THE ARUP JOURNAL

MARCH 1977

THE ARUP JOURNAL

Vol. 12 No. 1 March 1977
Published by
Ove Arup Partnership
13 Fitzroy Street, London, W1P 6BQ

Editor: Peter Hoggett
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Editor's note

In the Westmead Hospital article, which appeared in the October 1976 Arup Journal, the following credit was omitted.
Consultants to the New South Wales Health Commission:
Llewelyn-Davies Kinhill Hospital Planning

Front Cover: All Souls Church, Langham Place (Photo: courtesy Woodmansterne Publications Ltd.)
Back Cover: Computer drawing of Ebury Street Residential Development (Architects: Ted Levy, Benjamin & Partners)

All Souls Langham Place and the Waldegrave Hall

Poul Beckmann
Andrew Dekany

History

It all started when, after the battle of Waterloo, a large number of new churches were commissioned and Nash was developing Regent Street.

As John Summerson wrote in his *Georgian London:*

'Nash, with great wit and ingenuity, contrived to adapt a site at the top of Regent Street so as to provide his new street with an important terminal feature. All Souls, Langham Place, lies approximately at an angle of 45° to Upper Regent Street, but its circular vestibule, tangential to the church and placed on the axis of the street, closes the vista beautifully, and there is not the least suggestion of laboured compromise. It is really a case of the application of Nash's circus principle – the introduction of a circus when an abrupt change of direction is necessary – in town planning, extended to the plan of a building. The vestibule, with its effective Roman order and an upper stage comprising a spire rising out of a second colonnade, is amusing and unconventional, and the MP who, in 1824, said he "would give a trifle" to have it pulled down, was dull as well as spiteful.'

Nash, however, had worse tribulations to contend with than Philistine MPs. In a letter dated 14 September (1822) he wrote:

'. . . Regarding extra work in foundations, I found it necessary to sink them 3 foot lower, it being found intersected by old drains, sewers, cesspools and bogs in every part, and even now it will be advisable to take further precautioning to insure a secure foundation. A thin stratum of gravel half way down producing a considerable quantity of water has been cut through and must be prevented passing under the foundations – it will therefore be necessary to call in the aid of a greater surface of ground for the walls to stand upon. It is necessary to lay a course of stone flags 6 inches thick under the walls to afford a platform $7\frac{1}{2}$ foot broad for the walls to rest upon and also a drain in cement round the outer walls to remove the water from the gravel springs . . .'

Some of the entries in the records hinted at the way in which the difficulties were overcome and were to assume renewed importance 150 years later, as will be seen.

'8 February 1823

Payment for up to GL	main walls Reversed arches and piers for internal columns.

9 June 1823

Payment for	extra digging extra brickwork Yorkshire stone landings for under the footings of walls Drain in cement New drain in Riding House Lane'

After these setbacks work appears to have proceeded to completion in the familiar manner, punctuated by contractor's claims and with the heating being put in as an afterthought:

'25 August 1823

Payment for	building of tower above the height expected at this stage

26 April 1824

Payment for	glaziers and plasterers completion of work

22 June 1824

Flues laid under floor to warm the church.'

The next notable happening in the history of the building was in 1835 when gas lighting was introduced. This seems to have been a very controversial move as there are records of charges being brought against the Rector by a gas fitter for assault and battery! The introduction of electric lighting in the year 1900 appears to have been completed without incident.

By then considerable change to the interior had been introduced by the Victorians. We do not know Nash's original colour scheme, but it is more likely to have been a light beige or cream than the dark green which prevailed until 1940. The pews were originally painted to tone with the rest of the interior but mahogany pews were introduced in 1876. A large number of these were privately rented, thus providing the church with most of its income. These private pews were enclosed with doors, which were successively cut down to half height and removed by the year 1900 with the exception of one which remained until it was blown off its hinges in 1940.

In 1932 the BBC commenced its regular broadcasts of the morning service from the church. In 1933 the old vestry was demolished and the present Church House was built hard up against the east end, thus necessitating the blocking up of the two east windows.

In December 1940 a bomb destroyed the roof of the church and damaged the spire to the extent that it had to be partially taken down. Rebuilding started immediately after the war and the church was re-consecrated on 29 April 1951.

Situated where it was, the church had become a place to which many of the transient population of central London tended to gravitate. But the activities of the church, apart from the services, were badly hampered by the lack of a proper meeting place. The Waldegrave Trust allowed the use of their hall in Duke Street for three evenings a week, but there were no facilities for after service gatherings on Sundays, and the daytime meetings which the church wanted to form part of their activities, and this lack prompted several ideas to provide a meeting hall.

Demolition of the church and Church House, so as to allow redevelopment of the site, was considered briefly, but was rejected as being unacceptable apart from being beyond the foreseeable financial means.

Eventually the Waldegrave Trust announced that as their hall was getting old and was in such bad condition, they considered that they ought to sell the site for redevelopment in order to be good trustees of the assets committed to them. This meant that even the unsatisfactory facilities for meetings would be lost and served as a spur to the church to go ahead with the alternative project, which had been considered for some time.

In October 1971, Robert Potter had been appointed architect by the Parochial Church Council and it was shortly after that, that we became involved in the project.

We had first met Robert Potter during the design stage of the restoration of York Minster when Bernard Feilden had asked him to provide a second opinion on the proposed works. To what extent the undercroft at York provided inspiration for the new hall at All Souls we do not know, for Robert was at the time working on the conversion of All Saints' Church, Oxford, into a library for Lincoln College in which a new basement was to be created, but there is certainly a resemblance between the ways in which at both places extra accommodation was provided.

The idea for All Souls was based on Nash's mention of extra foundation depth, and entailed the excavation of the fill under the church floor to form a new basement to house the meeting hall, cafeteria, kitchen, etc., with the meeting hall having the new suspended ground floor over it spanning clear across the 13 m between the main nave piers. In addition, extra space for the congregation was to be provided by extending and remodelling the galleries.

Preliminary investigations

The first exploratory excavation by the south wall was carried out in May 1972 and revealed that the outside walls were deep enough to allow a basement to be constructed without necessitating underpinning of the main walls.

Thus encouraged, we wished to check the pier foundations, so a second trial pit was excavated inside the church, by the pier second left from the entrance. This pit revealed inverted arches as mentioned in Nash's records. Whereas Nash's reputation has tended to be that of a brilliant architect condoning shoddy workmanship, the brickwork here was beautifully executed, the mortar was hard and the bricks appeared of good quality. Further down the foundations were seen to step out in brickwork until eventually we found the York stone flags referred to by Nash, and these were thought to be wide enough to take the increased load from the new suspended ground floor without any strengthening. The District Surveyor was approached and inspected the gravel under the foundations and agreed that a maximum ground pressure of about 200 kN/m² could be allowed. A preliminary calculation indicated that this would be adequate, so that, provided the substructure found in the trial pit was representative of the whole, the scheme was feasible without major foundation works.

Meanwhile our investigations had revealed that the Bakerloo tube tunnels were very near the site and that one of the Victoria line tunnels passed right underneath the church, 15 m below street level. The rumble from the tube trains could be heard very distinctly in the vestry in Church House and could be faintly discerned in the church itself. Three questions were therefore raised: first, what would the noise and vibration be like in the new

Fig. 1
Excavation in progress seen through wide inverted arch at west end (Photo: Thomas-Photos)

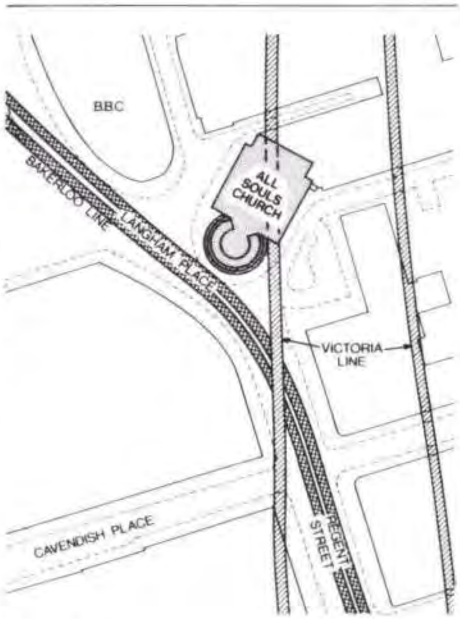


Fig. 3
Map showing position of Bakerloo and Victoria line tube tunnels in relation to All Souls Church

Fig. 2
Inverted arches under south aisle gallery (Photo: Thomas-Photos)

assembly hall? Second: would the noise level inside the main body of the church be increased by the creation of the hall underneath? Third: if the answer to either or both was yes, what could be done to mitigate the effects? We recommended to the church that they appoint Professor Peter Grootenhuis of Imperial College to advise on these problems.

Our recommendation was accepted and the Professor arrived with his complement of 'black boxes' and started to take measurements. A velocity meter was placed at the bottom of the internal trial pit on the gravel and on the foundation, and also on a ledge on the brickwork a bit higher up. For comparison, measurements were also taken in the vestry and in a boiler room in Church House.

The results were much as expected: the frequency of the noise varied between 20 and 40 Hz when trains started from Oxford Circus station and between 40 and 60 Hz when trains were braking on approach. The highest velocity recorded on the middle of the floor in the vestry was about 10×10^{-3} cm sec. This high level of vibration was partly due to resonance between the train noise and the vestry floor which had a natural frequency of 20 Hz. The velocity on the foundations in the trial pit was about 5×10^{-3} cm/sec. Road traffic was found to produce between one third and one half of the velocities recorded for the trains but with a lower frequency range, about 15–30 Hz.

The long and short of all this was that the noise in the basement hall would be disturbing for some of the functions envisaged to take place there, and the creation of the space under the floor of the main church combined with the fact that the new floor would be supported directly on the main piers and not as in the original church largely on dwarf walls in the fill, would lead to an increased level of train noise being heard in the main church. This latter noise increase was a very serious consideration because the BBC were using, and wanted to continue to use, the church for the morning service broadcast. The structural design for the new hall and the new church floor, therefore, had to incorporate measures to achieve the best practicable noise insulation.

The design of the new church floor

The new ground floor had to be raised 0.45 m above the level of the existing church floor, so that adequate headroom could be obtained in the basement hall without excavating below the level of the existing foundations, and hence without underpinning. This would reduce the headroom under the galleries but not unduly so, and from the point of view of the present-day use of the church, it was an advantage to bring that part of the congregation that would be seated on the galleries nearer to the main body of worshippers on the floor.

Even so we had to provide a structure which would span the 13 m clear distance over the hall with the minimum depth and with extra constraints on deflection imposed by the need for a natural frequency which had to be low enough to avoid resonance with train and traffic vibrations, but not so low that the floor would feel 'springy' or, worse, develop resonance with normal walking.

Robert Potter wanted to 'mirror' the inverted arches between the piers by making the ceiling over the hall a series of vaults, and this was considered also to be beneficial to the acoustics in the hall. The remaining design considerations were fire separation and sound insulation between hall and church, which combined to demand a minimum thickness of concrete of 100 mm and resilient pads under the joists or bearers for the new timber floor.

Our first proposal used pairs of welded steel plate girders approximately 500 mm deep with top flanges 100 mm × 35 mm and bottom flanges 200 mm × 20 mm. The narrow top flanges were to allow the placing of precast concrete vault segments that would span between the pairs of steel beams over a distance corresponding to the spacing of the main piers. The beams in each pair were to be spaced apart, so as to form a slot in which services and lights could be accommodated. This arrangement provided convenient spans for the timber floor joists which could be supported alternately on the top flanges of the steel beams and the crowns of the precast vaults.

For a variety of reasons it was decided that the vault segments ought to have diaphragms at their ends. This, together with the thickness dictated by fire and acoustical considerations, led to units which, to produce an acceptable ceiling pattern, would have to span 2.8 m, be 2.5 m wide and thus weigh over 2 tonnes each. We worked out ways and means by which these somewhat cumbersome units could be

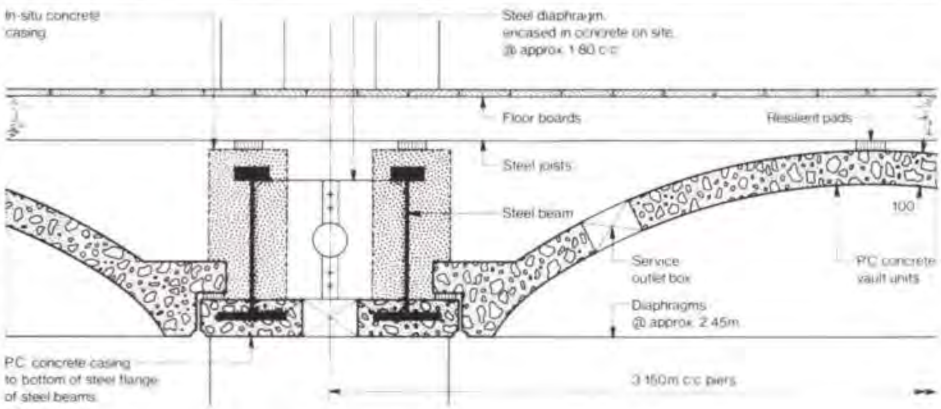


Fig. 4
Initial design of suspended ground floor structure

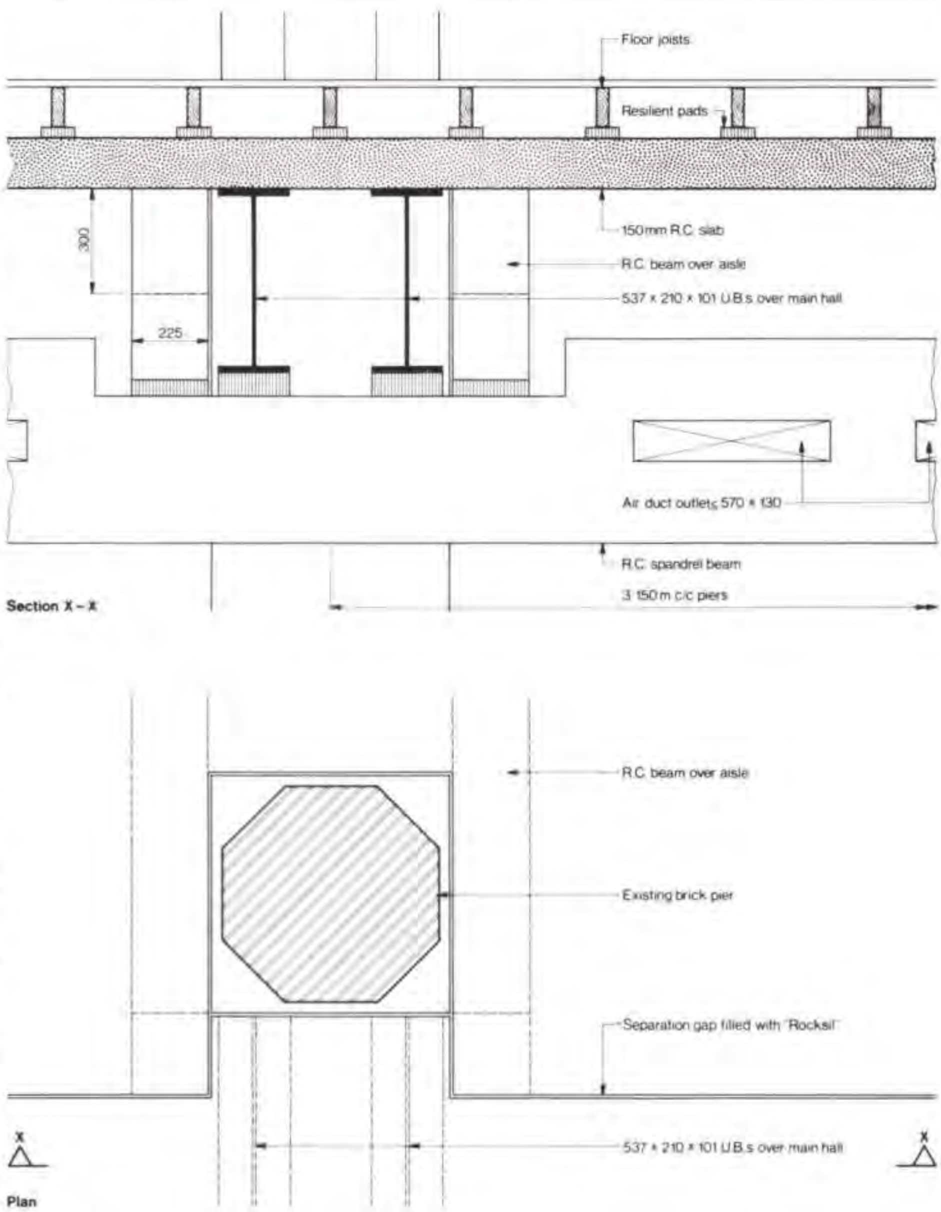


Fig. 5
Final design of suspended ground floor structure

unloaded (just off Regent Street), transported and placed, but the quantity surveyor's cost estimate combined with a number of technical detail problems to make us re-examine our ideas, the more so as the architect was uncertain that the 'as cast' finish would be acceptable.

The final design for the main floor over the hall consisted of pairs of Universal Beams 537 mm × 210 mm × 101 kg/m in grade 50C high tensile steel. These beams supported a continuous, reinforced concrete slab, 150 mm thick, on which the resilient pads for the timber floor bearers would rest. The vaulted ceiling would be formed in fibrous plaster and this had the advantage of allowing easier adjustment at the perimeter to accommodate the slight dimensional variations in the existing fabric, apart from providing a more controlable finish, and facilitating the fire protection of the steel beams.

The design change also simplified the transition to the structure of the aisle ground floor as this could now be a simple reinforced concrete slab and beam system.

Brick samples had been taken from the existing walls and cores were taken from the piers. The cores confirmed that the brickwork was well bonded even in the upper works, but the tests on the bricks indicated that even allowing for the hardness of the mortar, the brickwork whilst strong enough to support the loads which the new floor would impose, was not strong enough to allow the steel beams to be supported in pockets cut into the existing piers. It was therefore decided to build new brickwork piers 225 mm thick on the inside of the existing ones and join them with longitudinal spandrel beams which would provide support to the steel beams spanning the main hall and to the reinforced concrete beams which carried the slab over the aisles. The spandrel beams would also form closures to the false ceiling void in the aisles against the main hall. The new piers were supported on the existing foundations through reinforced concrete ground beams which distributed the load lengthways. In order to make the new ground beams act compositely with the existing foundations, it was decided to provide ground beams on the aisle side as well so as to sandwich the brickwork foundations between pairs of ground beams. To secure the connection, bolts would be inserted through holes in the brickwork and sleeves in the ground beams so as to allow them to be tightened up and provide a nominal transverse prestress.

Noise insulation

In order to isolate the church floor from the vibrations transmitted through the existing structure, both the steel beams and the reinforced concrete beams were supported on resilient bearings. These rubber and cork bearings would be dimensioned so as to give the whole floor structure a natural frequency as far as possible removed from the frequency of the train vibrations. At the same time the concrete slab had to be physically separated from all the existing masonry so that no vibrations were transmitted through contact that way. This separation could not be an open air gap as this would allow leakage of airborne sound as well as contravening the requirement that the floor should act as a fire break between the upper church and the lower hall. To overcome this difficulty the gap was filled by a sandwich of two sheets of *Asbestolux* with a 12 mm *Rocksil* mat in between. The sandwich was wired together with the *Rocksil* in compression and placed against the masonry before the concrete was cast. When the concrete had hardened and the formwork was stripped, the wiring was cut thus allowing the *Rocksil* to expand against the concrete, so as to fill the gap but allow almost free vertical movement.

Where the aisle beams were supported on the existing external walls of the church, pockets were to be formed, with concrete padstones as necessary, resilient pads of the appropriate 'tune' would be placed as bearings and the remainder of the pocket lined with soft material so that the beam end could be cast in situ into the pocket without creating a noise bridge.

For the basement the acoustical treatment had to be a compromise. Complete acoustical insulation would have required a 'box within a box' structure such as is used for broadcasting and recording studios. This was out of the question, on grounds of cost alone, in addition to which it would have been practically impossible to accommodate the required functions of the basement at the same time as making such stringent provisions. It was therefore decided to support the basement floor slab in the main hall through rubber pads on dwarf walls. The aisles, the entrance and the ancillary rooms at the back would have ordinary concrete floor slabs cast directly on the ground. The choice of pads for this floor was difficult as the unit area supported on any one pad was quite small and therefore provided only slight pre-load to the pads, which made it difficult to achieve the required very low natural frequency. The practical execution also posed problems but these were overcome by making the floor a composite construction of 65 mm precast *Bison* planks with an in situ reinforced concrete topping, 50 mm thick.

As the long span steel beams required a different stiffness of their pads from those of the aisle beams, the floor slabs were separated by *Rocksil* sandwiches from each other to allow for differential compression of the pads.

The stairs and the lift which gave access to the basement from the ground floor gave rise to a number of knotty detail problems as they had to connect parts of the ground floor supported on resilient bearings with parts of the basement floor resting on the solid. These problems were not made easier by the fact that often the obvious and (sometimes the only) place for the structural separation was one where a visual discontinuity was undesirable.

The gallery extension

The west gallery was extended in depth by one bay. The original gallery front which extended across the width of the nave had intermediate supports in the form of two double-width piers at about the third-points of the span. To have repeated this would have created two large 'blind' areas at the back of the nave; but even worse would have been the intrusion of the two new piers in the basement hall. A clear span therefore had to be provided.

This was achieved by a hanging Warren truss, which when clad, would form the new gallery front. The truss was spanning between two of the square-section brickwork plinths, which as parts of the side gallery fronts, formed the transitions between the octagonal piers supporting the galleries and the round shafts supporting the roof. The hanging form of the truss arose from the necessity to locate the bearings above the heavy continuous timbers supporting the side gallery fronts.

The brickwork in the piers was calculated to be strong enough to carry the extra load from the gallery extension. It was however not strong enough to allow the cutting of permanent pockets to house the truss bearings, so after these had been bedded, the pockets were to be dry-packed.

The gallery extension was thus given solid supports on the main piers, and hence no acoustic isolation from ground-borne noise. In this respect it was however no different from the existing galleries, and in any case, the timber construction under the gallery floor was considered to give a certain amount of damping.

Cranked universal beams provided the stepped support for the new gallery floor, spanning between beams on the line of the old gallery front and the new truss. Rigid joints connecting the cranked beams to oversize verticals provided the necessary lateral support to the compression boom of the truss.

The original panelling of the western gallery front was to be 'transplanted' to form part of the encasement of the new truss.

Durability

As always, when proposals are made for new work in old church buildings, durability was given special consideration. All structural steelwork was therefore specified to be hot-dip galvanized after fabrication, and the ties, connecting the new piers to the existing were to be of stainless steel. The bolts, which were

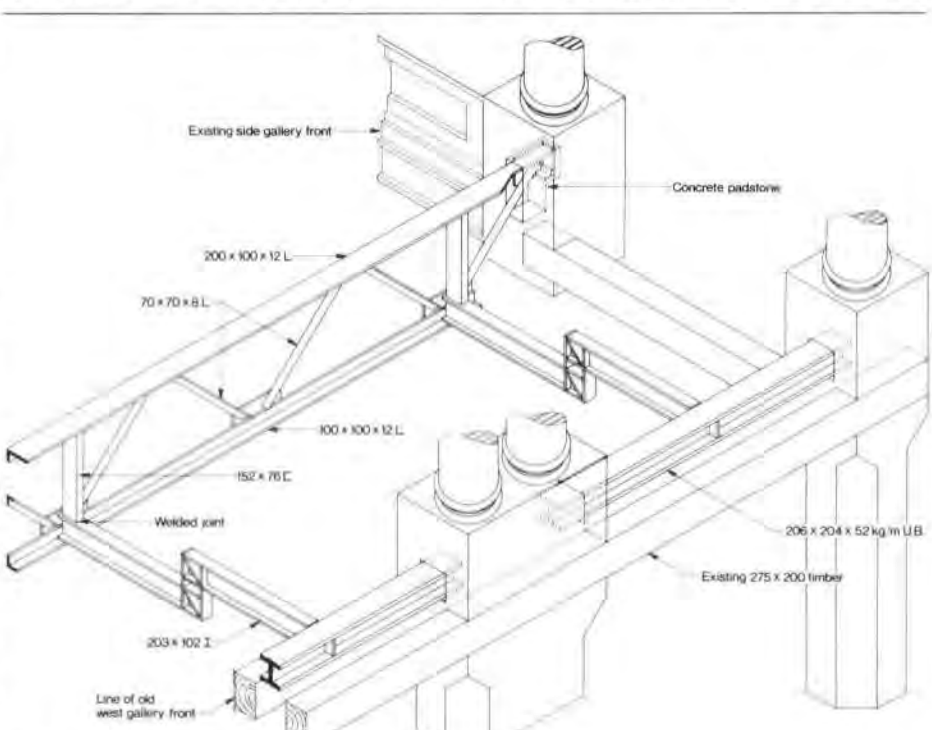


Fig. 6
Isometric view of new gallery structure

Fig. 7 above
View from new organ gallery towards the altar (Photo: Poul Beckmann)

Fig. 8 below
The enlarged western gallery with the rebuilt organ (Photo: Poul Beckmann)

to prestress the ground beams to the existing foundations, were to be wrapped in *Densotape* before their holes were grouted. This not only gave extra protection but also allowed unrestrained extension, when the nuts were torqued up.

Construction

Tenders were received on 3 April 1975 and the lowest bid came from J. W. Falkner & Son Ltd., who were just finishing the conversion of Holy Trinity Church, Southwark, for Arup Associates. Their tender 'including extra works ordered after completion of the Bills of Quantities' was £402,291 which together with the expenses involved in the temporary move of the church functions to St. Peters, Vere Street, was thought to be within the means of the PCC and it accepted.

Tenderers had been given the choice between a contract time finishing in October 1976 or a time of their own choosing, and had been asked to price each option. Falkners offered no reduction on an extended time and at the first meeting they showed everybody very clearly how they proposed to cut down what had always been thought of as a lengthy process, the excavation and disposal of the fill under the existing floor. Their proposal was to cut a temporary access opening in the north aisle wall towards Langham Street, and use this to allow a mechanical excavator to get into the church and carry out the bulk excavation. This site entrance was to become a major access into the building until such time as the new ground floor was complete.

No objection could be found to their proposal and thus we had the spectacle of a 'bulldozer rampant' inside the church. The excavation was, however, carefully carried out and finished off by hand. Gradually the inverted arches became fully revealed and were found to our pleasant surprise to extend not only along the full lengths of the aisles but also across the church at each end under the transverse colonnades. It was interesting to note that where the arches converged to form the pedestals under the piers, the mortar had been changed from lime mortar to Roman cement. Each pedestal was finished with a block of Portland stone, and from the offsets found at this level it was clear that the setting out had been checked and revised when the original construction reached this stage.

The fact that the external walls had inverted arches incorporated in them was intriguing, as there was no obvious reason for this feature. The thought occurred that perhaps the drawings might not have been entirely finalized when the foundations were constructed and that the builder was covering himself for the eventuality of the external elevations becoming a succession of isolated piers with tall windows between them. The excavation also revealed a reversed-curve 'bastion' protruding into the area which was to accommodate the main access stairs to the basement. This gave rise to some re-design of the stair arrangement. The next problem arose with the drilling of the holes for the bolts pre-stressing the ground beams against the brickwork foundations. Slightly optimistic from our experience at York Minster, where rotary percussive drilling had been found perfectly acceptable, we had not specified any particular limitations on the drilling method.

The drill used at York had, however, been a proper sledge-mounted, water flush, tunnel drill, which because the bit was kept in close contact with the bottom of the hole, gave rise to only low levels of vibration. It was not considered economical to use such a machine on this job and the hand held, air flush, percussive drills which were available through normal plant hire, shook the brickwork in a way which we did not consider permissible.

Unfortunately, the rotary drills which were available turned out to be very slow and hence some time was lost from the programme on this operation. After that, however, the ground beams went in without much trouble and the bolts were tightened to the prescribed torque.

The new piers were built on the inside of the existing, with flexible stainless steel ties giving lateral restraint and yet allowing differential shortening between the two lots of brickwork.

The spandrel beams had been designed as fairly slender but not unduly so, we thought. However, the requirement for 65 mm cover for fire reasons and the need to accommodate pairs of openings for ventilation ducts 570 mm × 130 mm made the reinforcement into something of a Chinese puzzle, and the thought of having to provide mesh in the cover to conform with bye-law requirements was definitely nightmarish. It was therefore agreed with the District Surveyor to use limestone aggregate which dispensed with the need for the mesh and the concreting was successfully completed by bringing in the concrete a little at a time and carefully vibrating each 'spoonful' as it went in.

The basement design envisaged the partial removal of some of the inverted arches, the twin ground beams being partly intended to compensate for this, and it was hoped that this demolition would provide bricks to carry out repairs which were seen to be necessary in order to realize the architect's intention of leaving the inverted arches exposed after grit blasting and cleaning. The strength of the mortar and the bricks resulted, however, in the bricks not coming away from the mortar in the way usually experienced in old buildings, and some repairs had to be carried out with new bricks.

The design of the main steel beams for the church floor had envisaged bolted joints at the third-points in order to facilitate the bringing in and the erection of the steelwork inside the building. The site entrance did, however, seem to offer the possibility of bringing the steel beams in, in one piece, traffic permitting (!), and this possibility was successfully explored and exploited by Falkners and the nominated steelwork sub-contractor, Warley Construction. Permission was obtained to close Langham Street over a Sunday, the beams were brought in and placed on the blinded floor. The rubber pads had been glued on to the spandrel beams in their correct positions before the steel was delivered, and the beams could therefore be lifted on to their final position by means of a 'stick' and this took place during the first half of the following week.

The concrete floor was concreted next in a judicious sequence of casting bays to ensure unimpeded access through the site opening, for as long as possible. This was during the middle of winter and although severe frost occurred, and the church felt bitterly cold, the shelter provided by the walls and roof proved adequate to ensure that concreting could proceed without any hold-ups due to cold weather.

The gallery steelwork did not pose any structural or constructional difficulties and we thought that structural site problems were at an end when we were asked to come down and have a look at the existing organ gallery. It had always been intended to renew the boarding of the gallery floor and it was whilst preparing for this that the site agent noticed a sag of some 50 mm in the two main timber beams supporting this part of the gallery.

Timber beams do, of course, deflect considerably with time, but the information that the new organ was going to weigh 15 tonnes, instead of the 13 tonnes of the old one, prompted a quick, very approximate, check calculation, which suggested a stress of 27 N/mm². This would have been a bit much even for best quality Pomeranian fir and these beams were certainly not first quality any more, having nasty splits in them. There was also an anything but confidence-inspiring connection to the primary timber, which had supported the original gallery front.

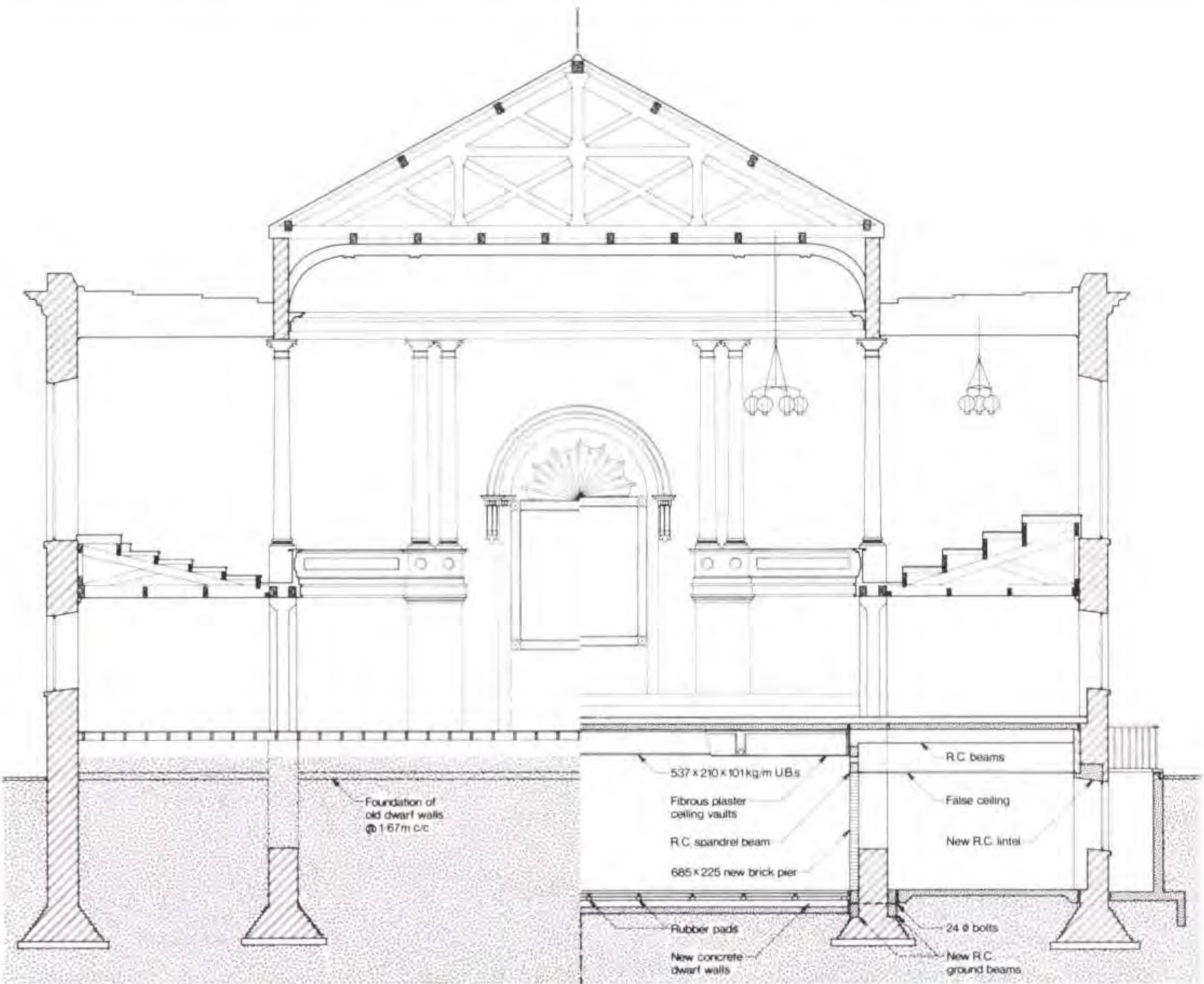


Fig. 9
Cross-section of church before and after refurbishing

In an attempt to minimize the cost of the remedial work, a scheme was first prepared for partial strengthening with steel plates, but when this was worked out in detail and a closer examination of the timber showed it to be in even worse condition than originally thought, it became clear that a complete replacement structure had to be provided. The original timbers had been 200 mm × 275 mm deep, and the replacement structure consisted of standard steel channels 254 mm × 89 mm, bolted to the sides of the timbers which remained in position, with the function of providing lateral and torsional restraint to the steelwork. The installation of this steelwork required the cutting of the eastern end of the two timbers under the organ in order that an 8 m long, 254 × 89 mm channel, strengthening the north-south timber beam, could be put into position. This meant a fair amount of temporary propping, as all the strengthening work had to be carried out with the floor and parts of the organ left in position. The timber beam was drilled for the necessary bolt holes using the steel channel as a template, before the connectors were inserted and the steel finally bolted to the timber. After that the similar operations could be performed on the east-west beams, for which the channels were supported in pockets cut in the brickwork of the west wall and, at the other end, connected by angle cleats to the north-south-channel.

This intermission delayed the organ installation considerably but from then on the only worry from our point of view was the maintenance of freedom of movement between those parts of the structure which were supported on flexible bearings and those which were solidly connected with the original masonry fabric. The bearings had been put in at a relatively early stage of the contract and it was inevitable that small accumulations of debris found their way to some of the narrow gaps surrounding the bearings. Another problem was that the screeds were laid afterwards and inevitably some grout spillage tended to bridge the movement joints in places.

Any bridging of the movement gaps would have formed a path for noise to travel from the tube trains via the existing foundations and piers to the parts of the structure which were supposed to be isolated from vibrations and if this were allowed to happen the considerable expenditure on vibration isolation would have been, at the least partially, wasted. A major 'mind the gap' campaign was therefore mounted and joined by the architects and the contractor. The success of the isolation measures has not been quantified but it would appear that the results are satisfying to the client.

In recognition of the contribution from the trustees, the new hall has been named the Waldegrave Hall.

The church was re-dedicated at a special service on All Souls Day, 2 November, 1976.

References

(1) SUMMERSON, J. John Nash, Architect to King George IV. Allen & Unwin, 1935.

(2) SUMMERSON, J. Georgian London. Revised edition, Barrie & Jenkins, 1970.

(3) INSIDE story. The building project of All Souls Church, Langham Place. All Souls Church, 1976.

Credits

Client:
All Souls Church, Langham Place

Architect:
The Brandt Potter Hare Partnership

Main contractor:
J. W. Falkner & Son

Quantity surveyor:
Wilson, Colbeck & Partners

Special anti-vibration pads:
Textile Industrial Components Ltd.

Fig. 10
The Waldegrave Hall: View towards the cafeteria, showing the original inverted arches, the new brick piers and the fibrous plaster vaulted ceilings (Photo: Poul Beckmann)

Fig. 11
The Waldegrave Hall: Close-up of glazed inverted arches, looking into cafeteria and showing ventilation grille through spandrel beam (Photo: Poul Beckmann)

Computer-aided drawings

Robin Whittle
Raymond Lee
Iain Lydon
Nigel Sherratt

INTRODUCTION

In the summer of 1973 the development of a computer-aided drawing system, CADRAW, began in Ove Arup and Partners. The function of the system is to manipulate, store and plot two-dimensional drawing information. During the past 18 months, it has been developed on our DEC 10 computer which incorporates multi-terminal facilities as well as batch processing. The computer configuration includes a memory size of 128K, 36 bit words, with two large capacity magnetic disks continuously accessible for storing and manipulating information. Two magnetic tape drives may be used to off-load long-term, infrequently-accessed information. A line printer (upper and lower case) and a 0.9 m drum plotter are directly connected to the computer and are kept supplied with automatically queued information. Batch running can be initiated by either a punched card reader or from the terminals.

This article gives a description of the system and how it works. It then goes on to explain how it has been used on three jobs and how this affected development work.

During the past year and a half, over 750 A1 and double elephant sized drawings have been produced and used for jobs (see back cover). Since it is a drawing system it does not just relate to buildings and reinforced concrete. It has also been used to produce schematic drawings for mechanical and electrical services. Other demonstrations include highway cross-sections and structural steelwork drawings, (see Fig. 1.)

It is very difficult to relate the cost of drawings by computer with conventional methods. However, the calculations show that the 750 drawings were produced at an average rate of just under two per man day. In addition to the number of man hours spent on data preparation, the computer costs, based on the current internal charging system, work out at an average equivalent drawing office cost of £14·50 per drawing.

GENERAL DESCRIPTION

The drawing components

System shapes

Drawings are produced by combining standard shapes from the SYSTEM SHAPE LIBRARY. This is a library of shapes commonly used on drawings. Fig. 2 shows pages from the Shape Library Catalogue showing examples of the shapes available. Each shape is numbered for reference. In order to reproduce it on a drawing the number of the shape is quoted. In addition its dimensions and the type of pen (i.e. thick or thin) must be given, together with the offsets and orientation of the shape relative to the intersection of the two grid lines or some other reference point on the drawing.

Job shapes – Macros

On each job there will be shapes that are repeated a number of times, but are not available in the System Shape Library. These shapes must therefore be constructed from the system shapes, but to go through the process of constructing one each time it occurs on the drawing would be very tedious. So the draughtsman can build up his non-standard shape once, and then keep it in the JOB SHAPE LIBRARY, which is a library of shapes particular to the job. These non-standard shapes are called MACROS.

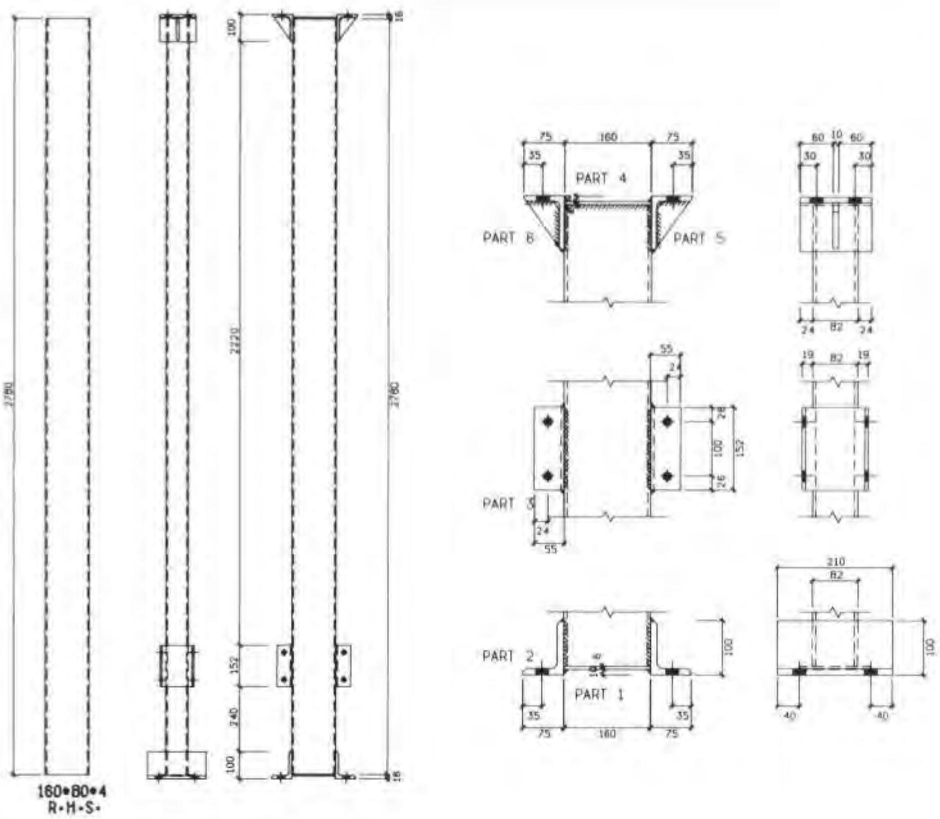


Fig. 1a
Structural steelwork – column details

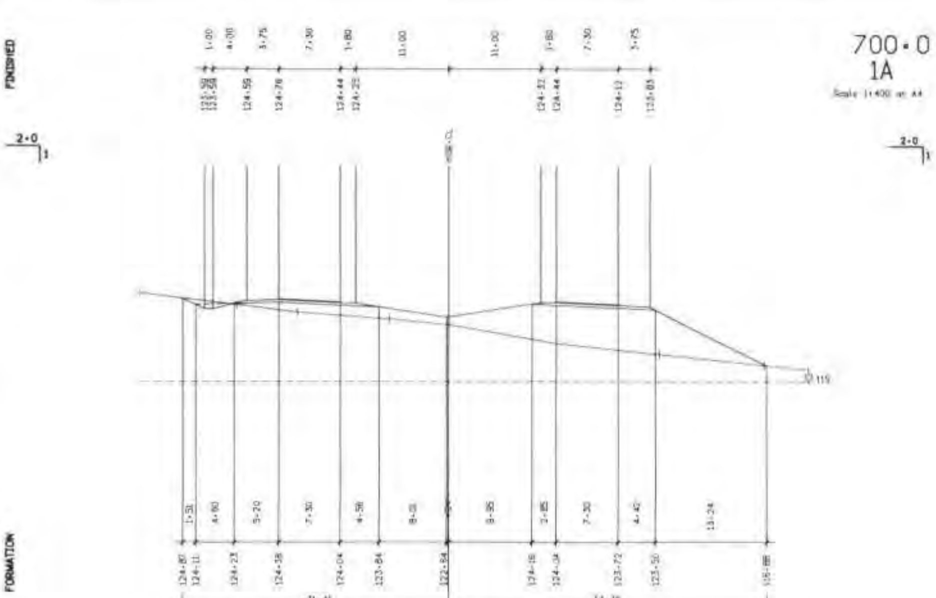


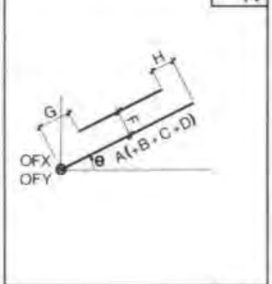
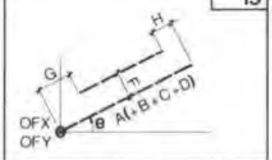
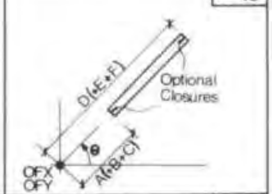
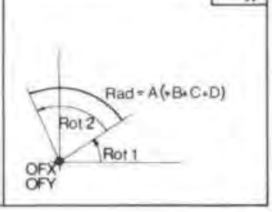
Fig. 1b
Cross-section through dual carriageway

Each Macro is given a title and a number by the draughtsman, for reference. Then it can be located on a drawing by specifying its number each time it is required, in a similar way to system shapes, the only difference being that Macros do not need dimensions or pen calls.

So by identifying a repetitive shape or area of a drawing and making a Macro of it, it can be reproduced by just specifying the number, thus avoiding unnecessary repetition of information.

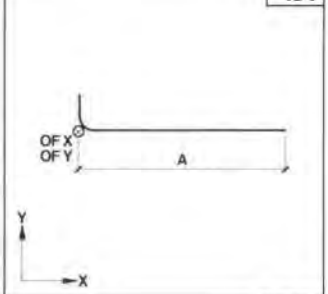
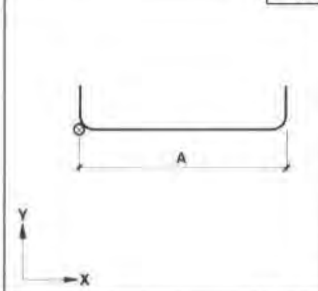
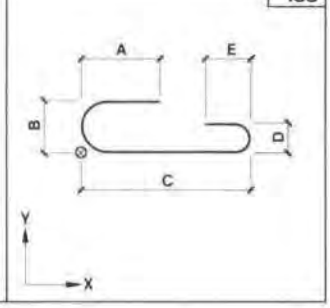
Fig. 3 shows how a Macro is built up and positioned on a drawing.

The use of Macros means that changes occurring in the design, e.g. a change in column size, can be easily incorporated and all subsequent drawings on which this Macro appears will automatically include the revised design.

Data Sheet Heading	Description	Sketch	Shape Number
OFX, OFY ROTATION A(+B+C+D as necessary) F G H	**WALL** Origin at Left Hand End of Line. Angle θ, from x-axis (decimal degrees). Length of line. Thickness of line. Length of dash. Length of gap.		14
	DASHED WALL As for subroutine 14.		15
OFX, OFY ROTATION A(+B+C) D(+E+F) G REF(1) REF(2)	**RADIAL WALL** Origin at centre of rotatation Angle θ from X-axis Inner radius Outer radius Thickness 'C' to close inner end 'C' to close outer end		16
OFX, OFY ROTATION LEVEL A(+B+C+D)	**ARC (see also shape 5)** Origin at centre of circle Start of arc ROT 1 Arc angle ROT 2 Radius of arc		17

Note: All dimensions in millimetres.
Pen size must be entered for each shape.
Handing is computed after any rotation: Enter 'x' about x-axis or 'y' about y-axis.

2a

Data Sheet Heading	Data Input Information	Sketch	Shape Number
OF X, OF Y ROTATION A LEVEL	As in sketch with no rotation and no handing. Angle θ, anticlockwise from x-axis measured positive in decimal degrees. Enter dimensions of shape in **mm**. Bar dia. in **mm**. N.B. Anchorage length dependant on bar dia.		134
OF X OF Y ROTATION A LEVEL N.B.	As for shape **134**.		135
OF X OFY ROTATION A, B, C, D, E LEVEL	As for shape **134**.		136

Note: As for shape **120**.

2b

Figs. 2a and 2b
The system shapes are the basic components or the building blocks of the drawing system.
The shape library is quite extensive and covers the *BS 4466* standard reinforcement shapes (Fig. 2(b)), text, dimension strings, and draughting symbols, as well as the normal drawing shapes (Fig. 2(a)).
The shapes are positioned on drawings relative to the grid line intersections, or a local reference point.

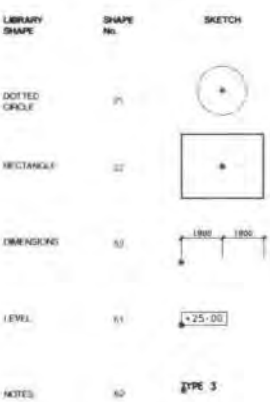


Fig. 3a System shapes

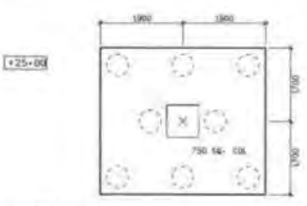


Fig. 3b Macro 703 (job shape) '8 PILE CAP'

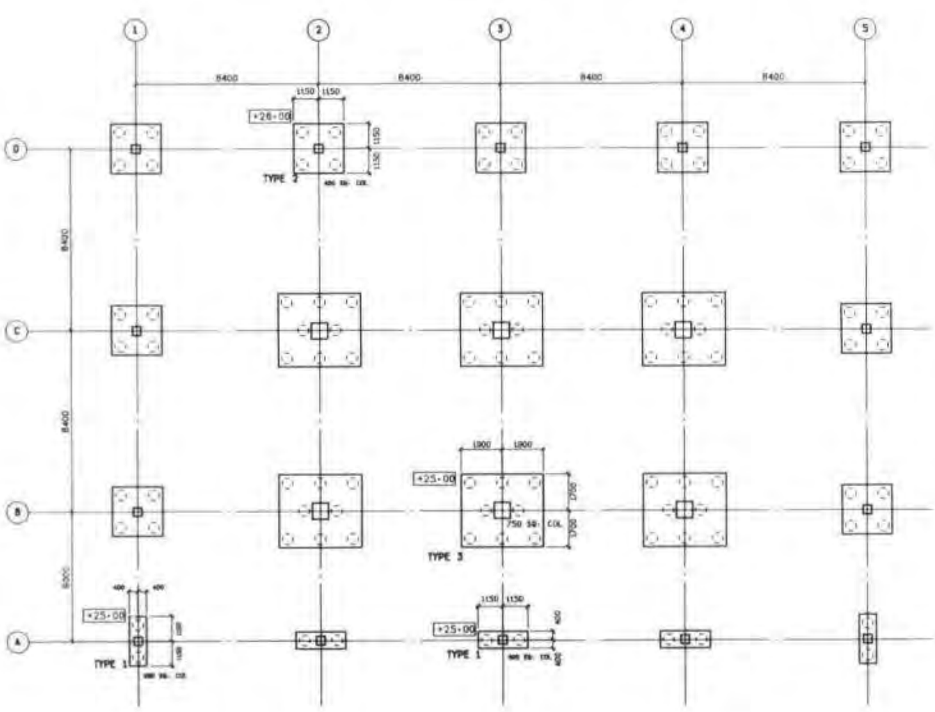


Fig. 3c Foundation drawing

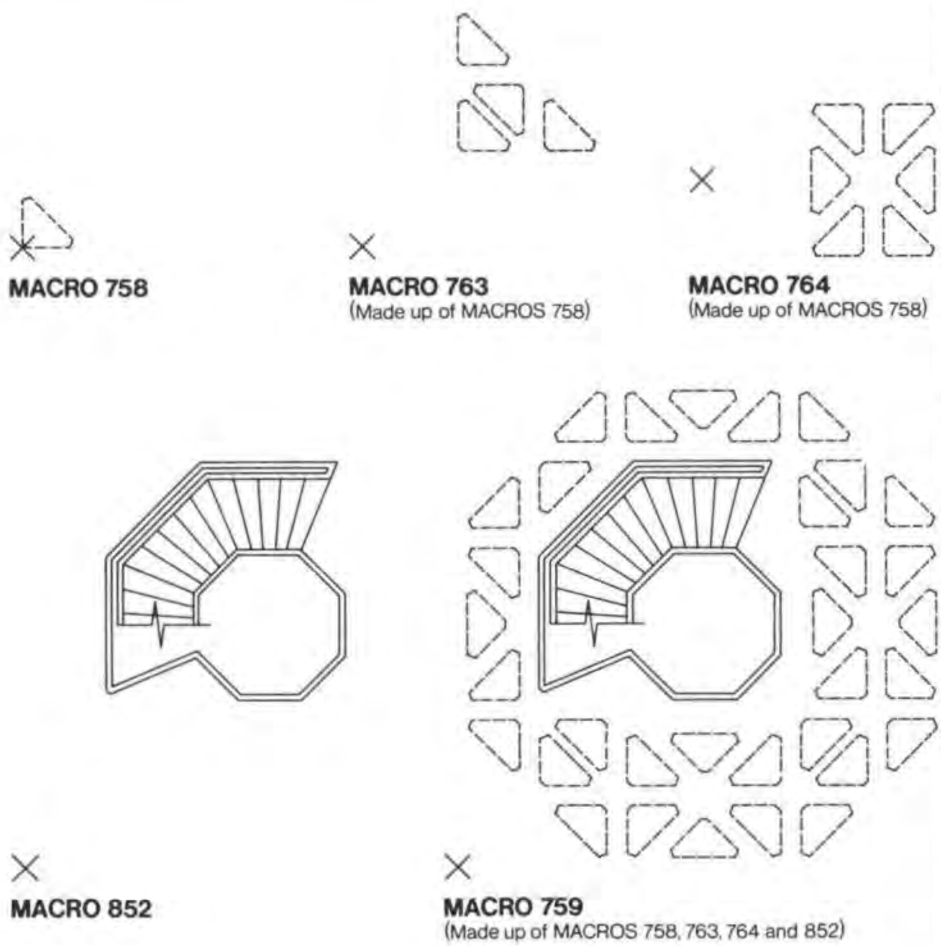


Fig. 4
Macros built up from others

Once a Macro has been created, it in turn can be used to build up further Macros. Fig. 4 illustrates this process.

Drawing production

Grid lines

At the start of a job an orthogonal grid system may be defined. This grid system is then used to position the information on a drawing. The grid lines are normally produced on the drawing, but they can be omitted.

Levels and overlays

Drawing information can be stored in levels, i.e. the information for a first floor plan would be stored separately from the ground floor information. This is the normal way of presenting layout drawings.

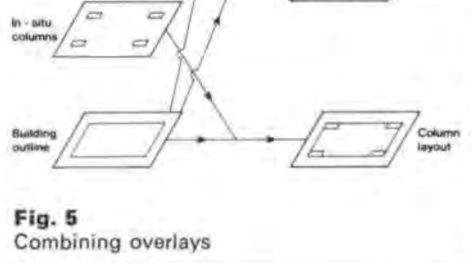


Fig. 5
Combining overlays

The CADRAW system allows drawing information to be further subdivided into overlays or sets of information. For example, the building outline, the precast units, the in situ members and the service information may all be stored as separate overlays or sets of information. By using the overlays in different combinations, the draughtsman can produce precast layout, services and general arrangement drawings, (see Fig. 5).

Producing the drawing and 'windowing'

If a grid system has been used then when all the drawing information is stored for a particular level, drawings of this level can be produced. To do this the area of the job to be drawn is specified, and the computer plots the drawing information in this area at the required scale. It ignores all the information outside the specified area. The process of specifying the area to be drawn is called windowing.

Drawings may also be produced if no grid lines are being used by positioning Macros and shapes relative to a drawing origin.

Zones

Normally, when a job is designed, it is divided into zones. In line with this the drawing information at each level can be stored according to its zone. However, a number of problems and difficulties were encountered when the drawing information was stored in this way. These problems are explained more fully later in this article. Suffice it to say that unless a job is particularly large, the simplest way to produce a drawing is to have all the information stored in one zone (i.e. the whole job) and to 'window' the area to be drawn. This window, or section of the job, can then be called a zone in the drawing title.

Organization of the drawing information

To produce computer drawings, the draughtsman must make a number of decisions at the outset of a job in order to make the most efficient use of the computer.

(a) What will be drawn by hand, and what will be drawn on the computer?

In the past when the decision to draw everything on the computer has been made, it has usually led to more trouble than it was worth. A balance must be struck between hand drawing and computer drawing – perhaps the computer is only used for the building outline, or even just the grid lines, or maybe it should do much more. The balance point will vary from job to job, and may be altered by current development.

(b) How will the information be stored?

This question is related to what drawings are to be produced. If structural, services and perhaps architectural drawings are required, then the basic outline would be stored in one overlay, and the information for each discipline would be stored separately.

(c) What range of scales is to be used?

This will affect how the information is to be prepared. Suppose 1/200 general layout drawings, and 1/50 detailed layouts are required. The drawings might be produced at 1/200, then using the same drawing information but with the Macro information altered to show more detail, the 1/50 drawings will be plotted.

JOB EXPERIENCE

Gulf University

The first phase of the university has been designed to take 2000 students and is positioned close to Doha City, the capital of Qatar. Owing to the intensity of heat and light from the sun in this part of the world, the design, prepared by the architect, Dr. K. el Kafrawi, is largely controlled by the need to reduce these effects. The result is a number of two-storey faculty buildings connected by covered ways with a number of enclosed courtyards, in which plants and fountains will be set. The layout is a honeycomb arrangement of octagonal modules, each based on an 8.4 m grid.

The work, valued at about £40m., includes the Faculties of Education, Science and Civil Aviation, library and cultural centre, audio and visual aids, a mosque, the administration buildings, an auditorium and student hostels.

The structure relies on load-bearing, precast panels with in situ concrete coffered floors. The outside walls are a double skin construction using precast concrete cladding panels.

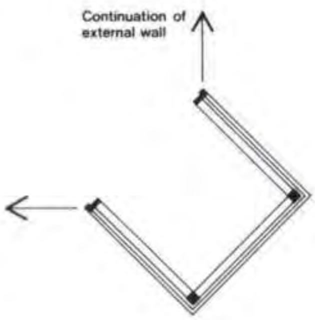
Drawing requirement

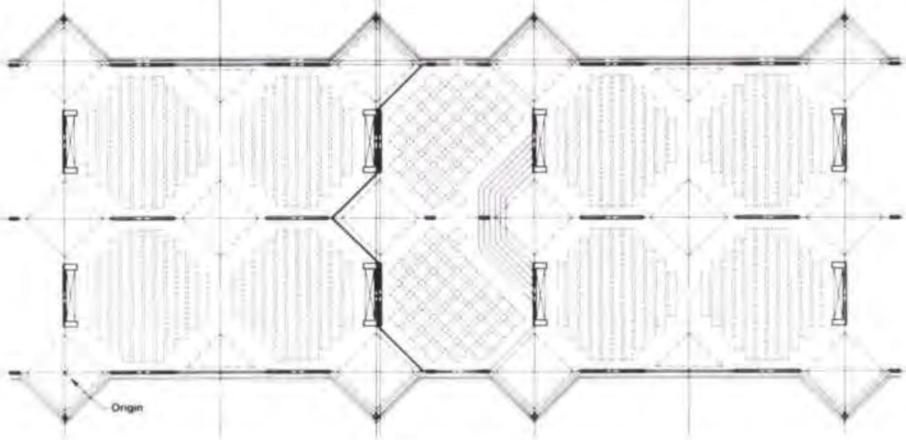
Initially it was intended that site work should start in July 1976. In the summer of 1975 the majority of drawings still had to be produced. A set of layout drawings was required early on for the quantity surveyors and later a more detailed set of layout drawings to act as key drawings for precast panels, sections and details for the tender documents. The first set was required by mid-September 1975 and the second by February 1976, but this date was eventually put back until the end of March. A third set, showing less detail, was produced for the architect.

Need for computer drawings

Early in June 1975, it was decided to give a trial run with the computer for one or two test areas. Although there was some doubt expressed about the quality of reproduction it was agreed that there could be considerable savings in time. Time was already becoming the controlling factor and the decision was made to produce the 1/100 and 1/200 scale layout drawings by this means. It was hoped that in this way a large number of draughtsmen, employed only for a relatively short time, would not be required.

This therefore provided the need to produce sets of layout drawings for four levels of the majority of the buildings by computer. These were foundations, ground floor, first floor and roof.





Figs. 6 and 7
Complex Macros

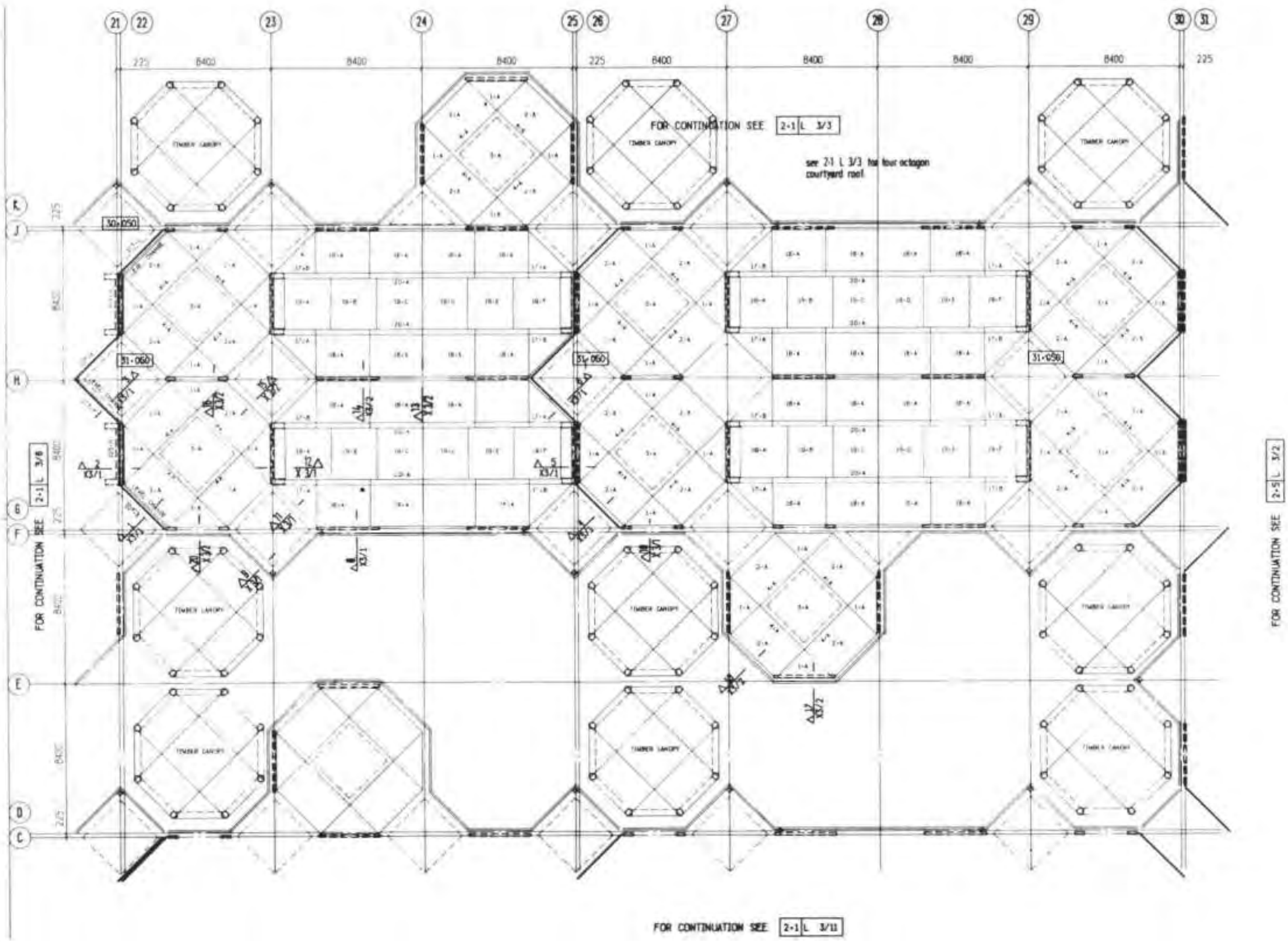


Fig. 8
Final drawing

Programme of computer work: Although it should be realized that much hand drawing work was going on throughout the period of computer draughting, there was a distinct difference in the type of work. Most of the hand drawings, early on, were of large scale details and many were used to develop the detail design. They were constantly being changed as new and better solutions were found. The first set of computer drawings, on the other hand, were required as quickly as possible and it was not vital that every detail should be correct. They were needed quickly to give the quantity surveyors something from which to measure.

During the peak pressure of work between late July and mid-September 1975 the number of people working on data preparation kept changing since it was also the peak holiday period. Over this period the average was just over two people and all were part of R & D. In this time 180 A1 drawings were produced. The two chief reasons for R & D doing the work at this stage were firstly, that there were no others available to do it and secondly that they were the people most familiar with the system.

All the drawings up to the end of October were produced by the IBM 1130 computer. This meant that all the data had to be punched on cards and entered into the machine in separated batches. Whilst this work continued the programs were also being converted to run on the new DEC 10 computer which was being installed at that time. For a period of six weeks both machines were being used and this allowed enough time to transfer the data for Gulf University and set up techniques to handle the new computer system.

Inevitably change in computers led to hitches and breakdowns but it was fortunate that the time coincided with a lull in required drawing production for this job.

The next pressure period for layout drawings started at the beginning of 1976. By this time most of the problems with the DEC 10 computer had been cured but only batch processing was available. Towards the end of this period of work the interactive program 'DREDED' became available and although this made a very significant change in terms of quick turn-round it came too late to make much difference to this part of the work. 209 drawings were produced in a period of three months. The data preparation was mostly done by people in Structures 3 with assistance from one person in R & D. A maximum of four people worked simultaneously, but for the most of this time only three people were working concurrently. As mentioned earlier the set of drawings produced in this period contained much more information than the earlier ones and in consequence required more time for data preparation.

Data preparation for layout drawings

The data preparation naturally divided itself into two parts. Building up each Macro was carried out by one person in R & D. The information for this was provided in the form of rough hand sketches or from some of the many hand-drawn details worked up by the experienced draughtsmen in conjunction with the engineers' and architects' requirements. Each floor had a different set of Macros but those at a particular level for one faculty were usually identical to those of another faculty.

Preparation of drawing data was carried out by two or three people in Structures 3. This consisted of calling up the Macros and other drawing data relative to the particular grid lines and giving them the correct offsets and rotations.

In order to reduce data preparation, some of the Macros were combined to make more complex ones. Fig. 6 shows one of these which is made up of four columns, three facing panels and three internal panels. Fig. 7 shows a set of floors and walls, used for the laboratories. The local origin of this complex component can be seen to coincide with a pair of grid lines.

The storing of panel type names was carried out semi-automatically. Although the system adopted proved very satisfactory for the roof, the number of practical difficulties for the other levels reduced the effectiveness of this method. The method adopted was as follows:

A special program was written which recognized the component numbers in the information already stored. This program scanned all the panel information and stored a separate overlay which contained the panel names positioned correctly. Both overlays were then included to produce the final drawing (Fig. 8).

A particular time-saving operation was carried out in producing the roof drawings. The information for first floor drawings had by then been stored. This was copied into the roof drawing store allocations. The information of some of the components was then altered so that whereas a floor detail was produced for the first floor drawings a roof panel layout replaced it for the roof drawings.

Another time-saving operation occurred in providing the architect with a set of drawings which were similar to the structural ones but without panel numbers, section and detail marks, and without any floor details. This set of drawings was produced with less than an extra man day's work.

One other important benefit from using the computer was from the rapid production of drawings once the data had been stored. 36 drawings were produced in one day during the more intensive time of production.

The final step to produce the drawings was to specify:

(a) The type of border required, the title and drawing number

(b) The notes required, both general and continuation drawing numbers

(c) Key plan of the faculty being drawn

(d) The required part layout drawing of the faculty.

Precast cladding panel detail drawings

After the layout drawings had started production on the computer, and the design team had seen the benefits of using a computer-aided drawing system, it was decided that production of the detail drawings for the precast cladding panels should commence.

In the early stages of the project it was decided that there would be one detail drawing per panel in order to reduce the number of errors which might occur on site when the panels were being cast. Also it was decided that a drawing of each panel, showing the geometric details, should be issued to the precast panel manufacturer as soon as possible so that the required moulds could be designed.

Each panel produced by the design team had some things in common.

(1) The exposed exterior face of the panel was a flat surface.

(2) The inside face had a ribbed structure which served both as a stiffener, and of locating the panels via cast-in sockets to the in situ structure.

(3) Each panel, depending on its adjacent panels and level at which it was located, had standard edge details.

The above similarities meant that the layout of each detail drawing would be very similar. It would consist of an inside elevation of the panel, together with horizontal and vertical sections and large scale details.

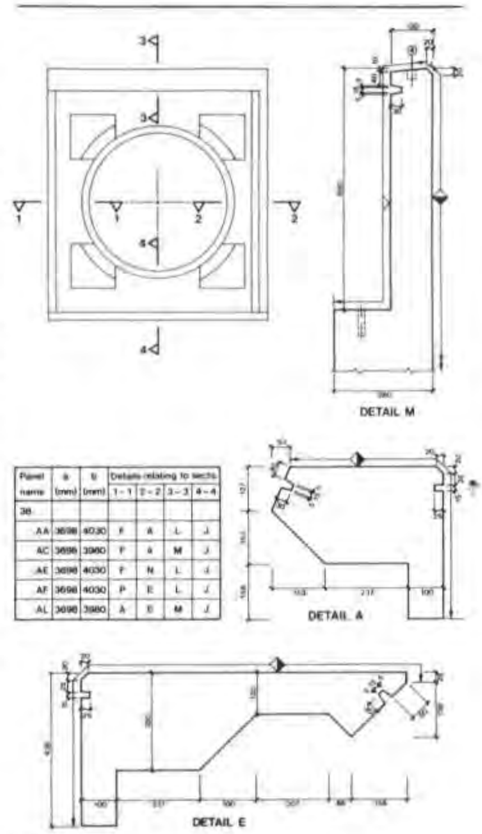
Once the engineer had designed a large number of the panels, they were rationalized in order to be grouped into families of geometrically similar properties. Sketches and tables were prepared for each family from which the final drawings were to be produced (see Fig. 9).

The first job for the draughtsman was to recognize the variations and similarities of each panel and to determine the basic details common to the family. (These common details varied from a circular window or rib structure to the overall dimensions of the panel.) Once this had been achieved it was necessary to establish what components were required to draw the whole family of detail drawings. At this stage the attitude of the computer draughtsman involved in this type of production work (a large number of repetitive drawings), tends to divert from that of the draughtsman who would have to do the same work manually. The manual draughtsman tends to approach each drawing individually. The computer draughtsman attempts to look at all the panels and decide how to minimize the information he needs to prepare.

Each completed section or detail Macro was a combination of a Macro of the linework and another of dimensions and titles. This meant that the linework Macro of a detail could be used again at a smaller scale as part of the linework for a section. Also it is envisaged that the reinforcement details for these panels will be produced at a later date and the linework Macros can be re-used again for these (see Fig. 10).

Once all the family component Macros had been created it was necessary to tabulate the Macro numbers required for each drawing. The final stage was to position each component Macro on the drawing sheet. Fig. 11 is an example of one of the final drawings.

Towards the end of 1976 an additional facility was built into DREDED which enabled a draughtsman to copy Macros from any other draughtsman's storage allocation into his own and use them quite independently. This optimization of use of the data stored meant that the time required for the input and creation of data was reduced.



Panel name	a (mm)	b (mm)	Details relating to sects 1–1	2–2	3–3	4–4
38						
AA	3698	4030	F	A	L	J
AC	3696	3980	F	A	M	J
AE	3698	4030	F	N	L	J
AF	3698	4030	P	E	L	J
AL	3698	3980	A	E	M	J

Fig. 9
Specification of panel details

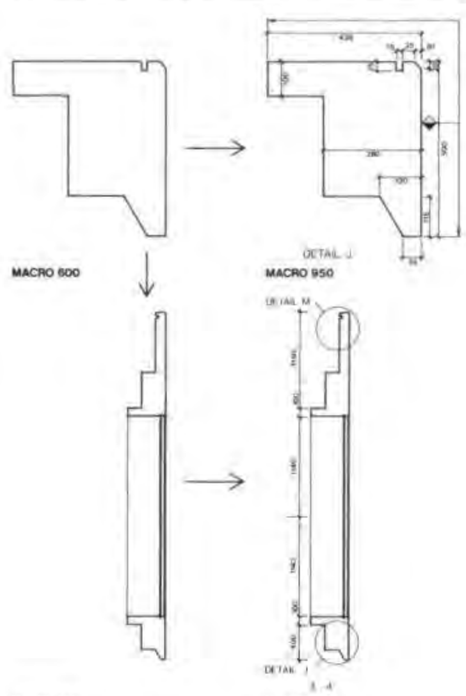
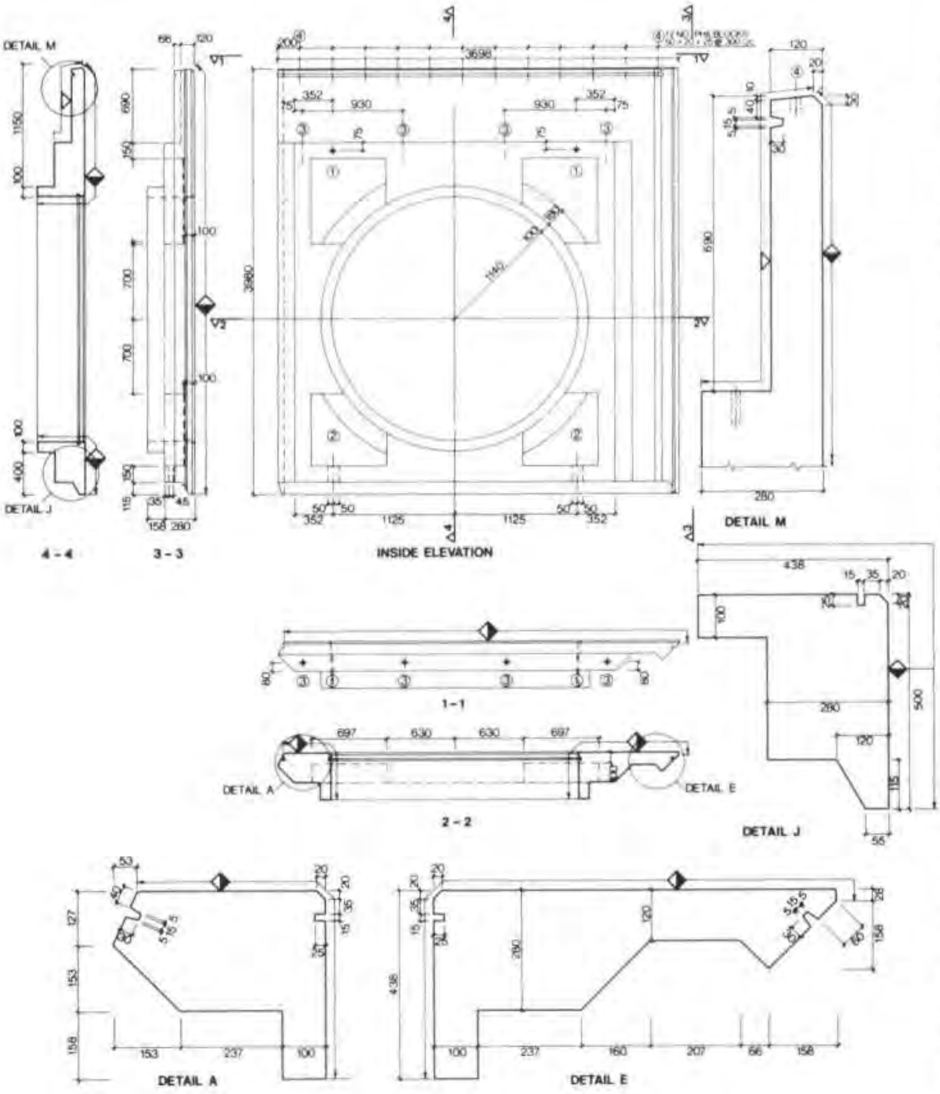


Fig. 10
Multi-use of detail Macros

One of the advantages of computer drawing was to show that since details are stored in terms of their real dimensions, any inconsistencies in the dimensions given in the sketches became apparent when the elements were assembled.

Problems encountered

A number of problems were encountered which were not easily solved. These divided themselves into those related to the performance of the computer and its peripherals, and those related to the program and the method of handling data. The problems discussed here only relate to the DEC 10 computer which was installed in October 1975.



LOCATION	4 LIBRARY AND CULTURAL CENTRE
TOTAL NO 38 AL	14

FIXING SCHEDULE (See inside elevation for item ④)

ITEM NO	NO. OFF/PANEL	TYPE OF FIXING
①	2	PFEIFER UNIVERSAL ANCHOR SOCKET TYPE NO 05 - 300 - 184 OR SIMILAR APPROVED
②	2	DOWEL POCKET TO RECEIVE 25mm DIA DOWEL
③	4	PFEIFER UNIVERSAL ANCHOR SOCKET TYPE NO 05 - 300 - 184 OR SIMILAR APPROVED

Fig. 11
Final drawing

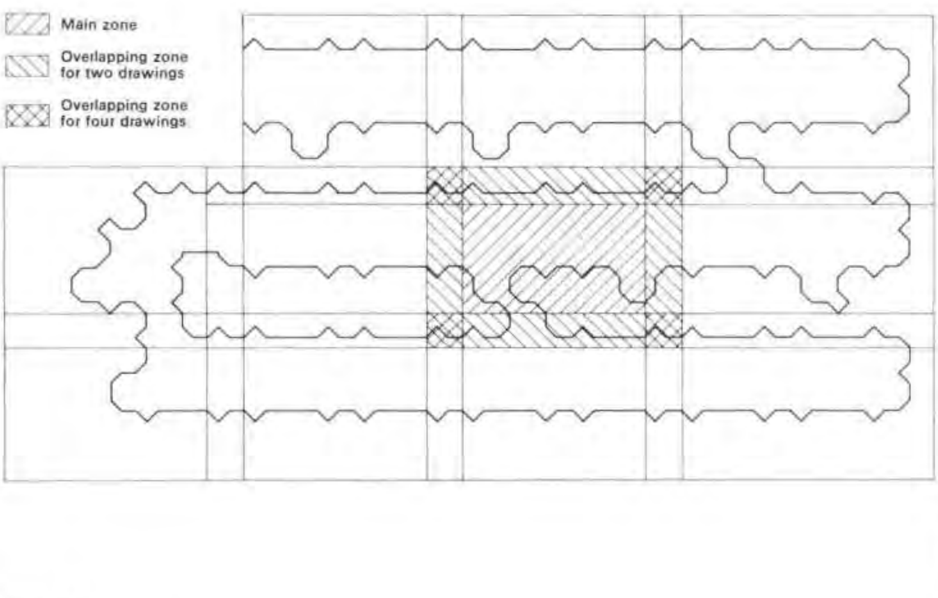


Fig. 12
Drawing zones

The first major problem of the computer was the possibility of losing the information. This could happen for a number of reasons, a common one being a fault in the electronic circuitry of the machine. This usually meant that, although the information could be reloaded onto the computer from magnetic tapes, the work done during the most recent day or two was lost. The number of people who were working during these days could be up to three or four, which could lead to the loss of several man days each time. Fortunately the faults of the machine in this respect were cured before the end of the job.

Another computer problem of major significance to the performance of the drawing system was the reliability of the one and only drum plotter. Early on in 1976 a breakdown could mean two or three days delay before the engineers appeared to correct the fault. In order to improve the situation, a number of the more frequently used spares were kept within the premises and fitted as soon as the fault occurred. This reduced the time out of action to a few hours. In spite of these efforts to improve the reliability, the quality of drawings was not always satisfactory. Lines sometimes did not show, either because one the pens dried up or because it was set slightly too far from the paper. This meant that the drawing had to be replotted.

Although care in handling the plotter by the operators helped to solve this problem, it remained to the end of the job. Since the spring of 1976 a second plotter of the same make, but a later version, has been installed 'on trial'. This has proved to be no more satisfactory and further trials of other brands are pending.

There were three major problems related to the program and the handling of data which prompted further development of the program. The first related to the overlapping of layout drawings. This was required to ensure that all details were shown on one drawing at least and did not get missed because they occurred at the join of two drawings. In order to achieve this overlap and yet still only store the information once, a complicated process was required. Each faculty was divided into a number of zones, as shown in Fig. 12. A particular drawing would include not only the large internal zone but also up to eight overlap zones. This led to a number of mistakes being made and much frustration in correcting them. An example of this was that a particularly large component, if stored in an overlap zone, stretched well into the adjacent one. Hence it could either be split up so that for this particular situation only part was stored in the overlap zone or it could be stored in the adjacent one, in which case it would appear fully on one drawing and not at all on the next. The former caused extra awkward work, and the latter led to mistakes being made in preparing data and remembering in which zone each component was to be stored.

Although this was chiefly just a frustrating problem it did point to a severe restriction of the program. It was this that prompted some new development work for 'windowing'.

The second major problem was concerned with placing text, section and detail marks on a drawing. Usually these are positioned approximately on hand drawings but care is taken to ensure that overwriting of other parts of the drawing does not occur. To do this by data sheet requires much care and effort and even then mistakes are made. The development of the interactive program DREDED helped towards correcting mistakes quickly but it did not reduce the initial effort. It is in this area that the system requires further development and this is discussed at the end.

The third major problem was concerned with drawing complicated shapes. The technique available for most of this job was by connecting a series of straight lines, each of which required x and y offsets for the start position and a length and angle from the x-axis to set it out. 'L' and 'U' shapes could also be used where appropriate; however it was a tedious way of entering data.

During the early part of 1976 a new system shape was added to the program known as the 'Walk Round'. This enabled the data preparer to start from one point and, by giving subsequent x and y dimensions, could cause a continuous line to be drawn changing direction as required. Extensions to this theme were to draw full or dotted lines in either pen size and draw single, double or four parallel lines.

This system shape was used a great deal for the latter part of this job and was especially useful for creating service duct routes for the foundation and ground floor drawings. Unfortunately the development had not reached the point where junctions and corners could be specified. These had to be added as separate Macros.

Lloyd's, Chatham

The new administrative offices and computer installation for Lloyd's of London was designed by Arup Associates Group 7. The site is at Chatham on the River Medway, once part of the Naval Dockyard. The building has five floors with a total area of 18,600 m², its estimated final cost being £8 m. Precast concrete units are used for the structural ceiling and supporting columns; the rest of the structure is cast in situ.

Work started on site in October 1975 and completion is programmed for June 1978. Production of computer drawings started in June 1975 and finished in December 1976. During this period 170 drawings were created, at scales of 1/200, 1/100 and 1/50. The drawings were produced by Robert Myers, in addition to his normal design work, with occasional assistance from the CADRAW development team. The Arup Associates' architects and engineers produce their own drawings, so the use of an engineer's time in preparing drawing information, as well as the initial hand sketches, is in accordance with normal practice.

The final drawing office cost, based only on computer charges, was £12 per drawing. This was one of the first jobs to have a substantial computer drawing content, and a great deal of useful feedback was obtained which led to important improvements in the system.

Drawings produced

All the drawings had a double elephant border, which was added to the system library especially for the job.

Five types of layout drawings were produced to show:

(1) General arrangements at 1/200 scale

(2) Precast elements at 1/100 scale

(3) In situ slabs at 1/100 scale

(4) Reflected ceiling plans at 1/100 scale

(5) Blockwork at 1/50 scale.

These drawings were created from varying combinations of six overlays:

(1) In situ columns

(2) Slabs

(3) Floor plans (not showing all the structural elements

(4) Precast columns

(5) Precast ceiling units

(6) Ceilings (reflected plans not showing all the structural elements).

One drawing was sufficient to show each floor at 1/200 scale. For the 1/100 scale drawings, which formed the majority, each floor was shown on four drawings.

Windowing

Windowing trims the drawing information, so that only a predefined area of the building is plotted. At the start of the job, windowing was not available and the information had to be divided into zones which at the largest scale would fill one drawing. Since although zones could be combined (for a drawing at a smaller scale), they could not be subdivided (for a drawing at a larger scale).

Initially no 1/50 scale computer drawings were to be produced, so the floors were divided into four main zones for the 1/100 drawings. Since a string of identical elements can be stored almost as quickly as a single element, this division of the drawing information was particularly frustrating when large numbers of identical units covering the whole floor area were required, such as the 'spider' shaped precast ceiling units in Fig. 13. A further complication arose when storing elements, such as columns, which occurred in areas common to adjacent drawings. If these columns were stored once for each 1/100 scale drawing in which they occurred, then any alterations to the columns meant editing two sets of information. Also, in the 1/200 scale general arrangements, when the four zones were combined, the columns were drawn twice, causing some disintegration of the paper and consequent clogging of the plotter pens.

An initial solution to these problems was to store the common information in 'overlap' zones. Thus each floor was eventually subdivided into eight separate zones. This plethora of zones made the editing of information, which occupies the major proportion of time spent on producing drawings, unnecessarily complicated and time-consuming. At this stage, some 1/50 drawings were requested, and the prospect of re-storing the information, divided into 40 zones for each floor, prompted the development of windowing.

After the introduction of the new facility, the drawing information was stored in one large zone for each floor, and trimmed by the program to produce the drawing areas required.

The increase in computing charges caused by examining more information was £0·50 (DOC) per drawing, no extra plotting time being required. This increase in cost was amply justified by the consequent reduction in man hours needed to edit drawing information.

Ceiling unit Macros

The advantages of storing the drawing information as Macros can be seen from the case of the precast ceiling unit. This unit forms the services void, and is a major element of the structure. When work started on the computer drawings, the design of the unit was not complete.

Wherever the ceiling unit occurred in the drawings, it was called up by its number in the job shape library. Thus to keep the drawings up to date as the design developed, it was only necessary to alter the information stored under that Macro number, rather than having to alter the ceiling units on the layouts. This method made it possible to produce new layout drawings overnight, enabling the group to proceed with the design without delay. The drawings were updated three times in this way.

Services

The services drawings were produced by combining computer and manual draughting. This combination proved to be effective, and was used for other drawings. Copies of the structural drawings were produced on the computer in red ink, to which the services were added by hand in black. By using two inks, the services information was printed boldly, with the structure as a fainter background.

Mirror image drawings were produced by the computer, on what became the back of the sheet, so that none of the original work was erased during the addition of the services. No alteration to the stored information was necessary.

Production of drawings

To produce a drawing it was initially necessary to specify:

(1) The job number and title

(2) The drawing number and title, and the type of border

(3) The area to be drawn and the scale

(4) The level, and the overlays of that level to be combined in the drawing.

In a set of layouts for the whole building, much of this specification was common to all the drawings. A means of reducing this repetition was devised.

The specification for a typical floor was stored.

A small subsidiary program altered this specification to produce the drawing required. To assemble a drawing, it was then only necessary to specify:

(1) The area to be drawn

(2) The level

(3) The drawing type.

Using this method, a high rate of production was achieved.

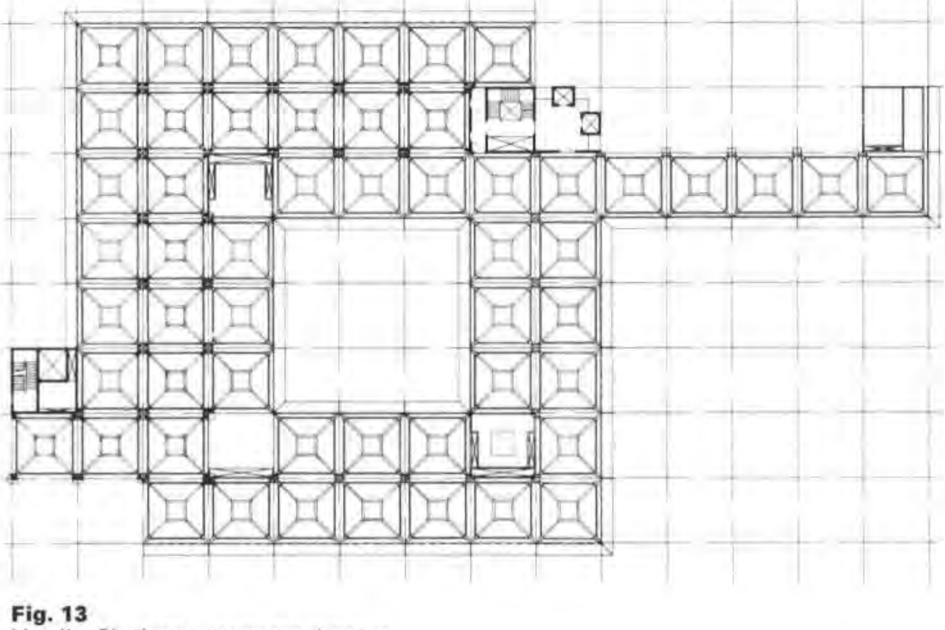


Fig. 13
Lloyd's, Chatham – computer drawing

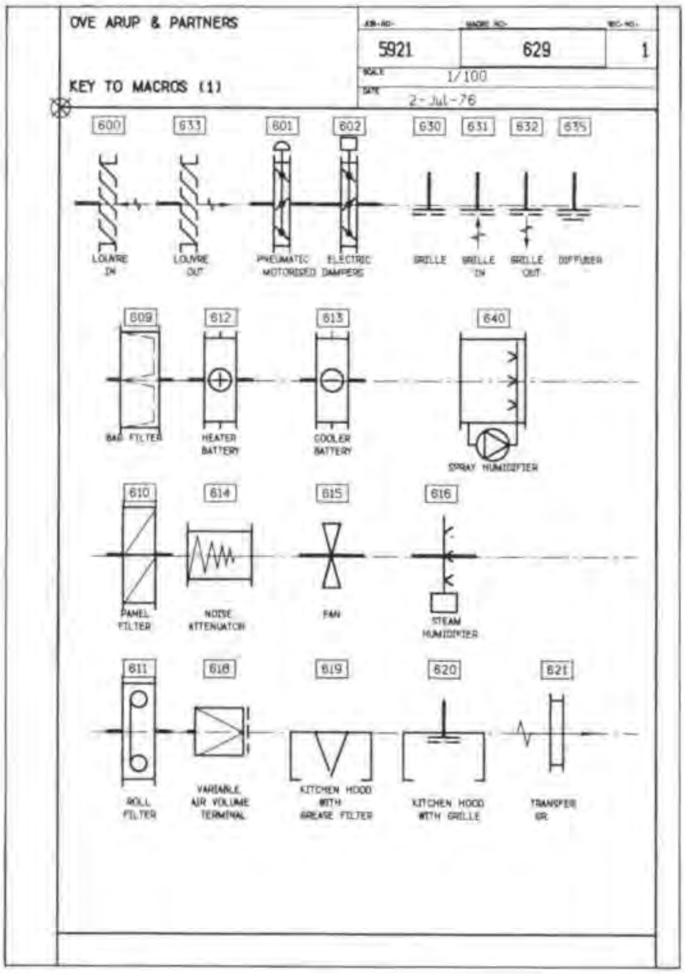


Fig. 14
Library of mechanical/schematic Macros

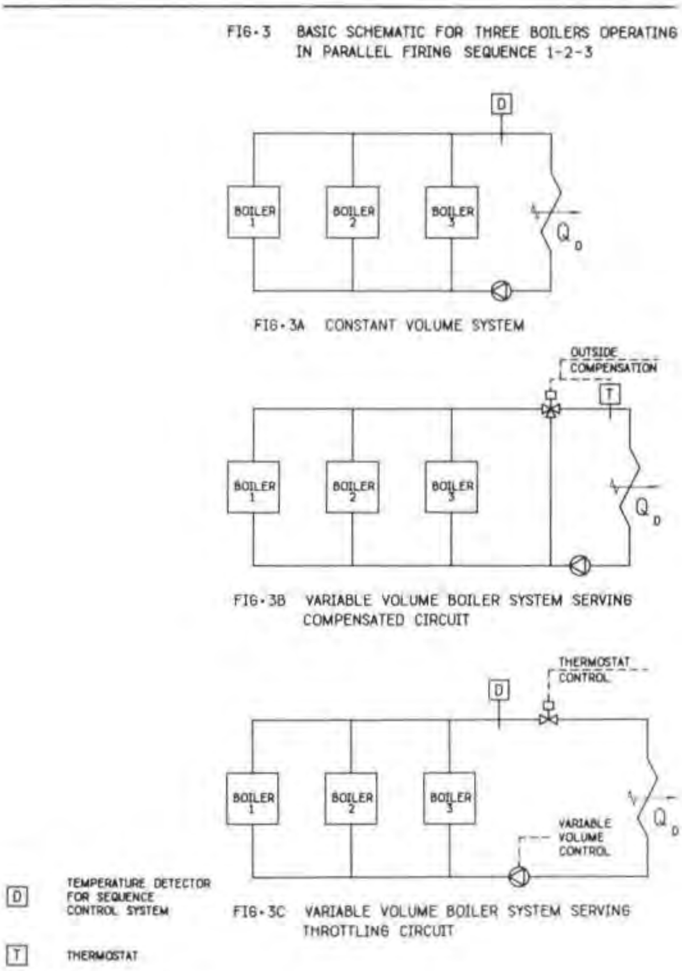


Fig 15.
Services schematic drawing

Portsmouth Dockyard Offices

The No. 3 Basin redevelopment in Portsmouth Dockyard includes a three-storey office block. This building is intended to house the office staff, who will administer the activities in the adjacent dry docks, and also contains workshops and a retail store at ground level. It has a total area of 6000 m² and its estimated final cost is £1.8 m.

This job is being designed by Arup Associates Group 1, and the requirements for drawings are similar to Lloyds, Chatham, in that the different disciplines within the group require variations of the same drawings. The computer drawings were produced by Derek Pike with assistance from the CADRAW development team. These drawings have formed the basis for both the structural and architectural drawings for the job and consisted of the structural outline, column details and layout of the coffer floors.

In the late summer of 1976, 46 drawings were created at scales of 1/100 and 1/50. The drawing office cost, based only on computer charges, was under £5 per drawing. In addition to this the time spent in data preparation was minimal. The initial setting up of data took half a day and the consequent corrections and alterations took less than a man week.

Probably the greatest benefit was due to the use of the 'repeat' facility. One coffer was positioned and then repeated a number of times in one direction to make a Macro containing a line of coffers. This line of coffers was then repeated a number of times in a direction at right angles to the line. This created a large rectangular area of coffers from three lines of data information. Furthermore one instruction could change the coffer outline from a dashed line to a full line. This was required in order to change the structural layout drawings to reflected ceiling plans.

Schematic drawings for Mechanical and Electrical Services

The application of the CADRAW system towards producing schematic drawings has had very limited use so far. However it has been demonstrated that drawing data can be stored very rapidly.

In order to achieve a simple and rapid means of producing drawings a library of specialized Macros was required. Fig. 14 shows a part of this library. Each of the components shown is stored in the computer and can be used in a similar manner to any other drawing Macro.

The method of producing a drawing divided itself into firstly placing the components in their correct positions on a drawing and secondly joining them together with lines representing ducts or piping. The first part was achieved simply by writing on a preprinted grid plan the component numbers at the required spots. This information was typed straight into the computer. The second part and any text required were entered onto data sheets and then typed into the computer or punched onto cards.

There has been one difficulty in producing drawings this way. This is the amount of effort and care to place all the connecting lines correctly. The use of the interactive program, DREDED, provides a rapid means of correcting errors, but by then it is too late. In order to overcome delays and waste of the users' time the sequence should be as follows:

(a) Prepare the data for all the components which make up the drawing.

(b) Produce a picture of this on the Tektronix screen and make any corrections necessary.

(c) Create all connecting linework directly on the screen and add any notes required. These operations take very little time and can be seen to be correct.

(d) Store the information and produce the drawing.

The time taken for the whole of this operation is not likely to be more than two hours.

FUTURE DEVELOPMENTS

Screen editing

Development work is under way to provide screen editing facilities using the Tektronix storage tubes. This will open up a greater potential for computer usage. The two main types of drawing work which this concerns are:

(a) Placing text quickly and correctly anywhere on a drawing

(b) Creating or editing linework directly.

One way of doing this is by manipulating the cursor lines, which appear on the screen, with the control wheels on the key board. This gives immediate response to the user and he can see if the result is correctly positioned and not overwriting other information.

Hewlett Packard 9830

The Hewlett Packard 9830 desk top computers are used in four of the regional offices. They are not only useful computers for drawing work in their own right connected to small table top plotters, but can act as terminals to the DEC 10 computer in London.

Programs have been developed on the HP 9830 so that components can be plotted on its plotter and corrected as necessary. The data for a number of these can then be transmitted and stored on DEC 10 using the HP 9830 as a terminal. The assembly of components and other drawing data and the production of drawings produced on the DEC 10 plotter can then be carried out using the HP 9830 as a terminal. Sets of grid lines can also be set up on the HP 9830 and checked before transmitting to DEC 10.

Bush Lane House fire protection by water cooling

Turlogh O'Brien

Introduction

Water cooling is still sufficiently novel a method of fire protecting structural steel for it to be useful to explain some of the background in order that the details of its application to Bush Lane House can be appreciated. Although it is usual to refer to the patent taken out in 1884 by Wright for a system of water cooling cast iron columns, the main developments of the idea have occurred in the last 10 years.

Tests to explore the validity of the technique were initiated in the mid-1960's in England by Stewarts and Lloyds (now BSC, Tubes Division) at the Fire Research Station, in Germany by E. Knublaunch, and in Australia by Improved Constructions (NSW) Pty. Ltd. at the Commonwealth Experimental Building Station. Among other things these tests showed the difficulty of doing meaningful tests on water cooled systems, as will be apparent later.

Meanwhile the Americans, using a theoretical approach developed by L. G. Seigel of US Steel Corporation, proceeded to apply the system to a very large project, the headquarters building for US Steel in Pittsburgh, completed in 1971 (architects: Harrison, Abramovitz and Abbe). The Seigel method has subsequently been used as the basis for the water cooling systems on a number of further buildings in various parts of the USA.

In Europe, the first project completed using the system was the refurbishment of the SNCF offices in the Champs-Elyseés, Paris (architects: Forestier & Goldfinger) completed in 1969. Since then only a very few other buildings have incorporated the method, the most notable being the Operations Research Institute of the German Iron and Steel Producers Association, Düsseldorf (1970).

Bush Lane House is the first building in the UK to use the method. In the design, although careful attention was paid to the previous work, particularly that of Seigel in the USA and Mommertz in Germany, there are features which are unique and which merit description.

The basic principles

Structural steelwork is normally prevented from losing its strength in fires by applied insulation, so that its temperature does not exceed a critical value, often 550°C. In water cooled systems the aim is to remove the heat reaching the steel sufficiently rapidly that its temperature does not rise too high. Removal of the heat is achieved by raising the water temperature (specific heat) but more effectively by evaporating it (latent heat). Whilst 1 kg of water absorbs 314 kJ for a 75°C temperature rise, it absorbs 2150 kJ whilst evaporating.

Heat is readily conducted from the steel to the cooling water and it has now been shown by calculation and by tests that as long as water remains in contact with the steel throughout the fire, its temperature will not exceed critical values, except under very unusual circumstances (e.g. exceptionally thick steel). The problem in designing a water cooled system thus becomes one of making use of the latent heat of evaporation of water whilst ensuring that water is always in contact with the steel that is exposed to fire.

The essential elements in a water cooling system are thus:

(1) A means of separating steam from water and venting it safely to atmosphere

(2) A means of maintaining the head of water in the system, making good evaporation losses

(3) A means of ensuring adequate flow of water to all parts of the system avoiding unintentional steam traps.

Subsidiary considerations include the provision of suitable facilities for filling and draining down, treatment of the water to prevent freezing and corrosion, and provisions for maintenance.

In the majority of completed buildings the technique for achieving these ends has been to interconnect the structural members with pipework so that in the event of fire the water circulates and steam is separated in a tank on the roof, which also serves as the reservoir to replenish the system. This type of design may be described as a circulatory system with replenishment. Because circulation of the water occurs, a large quantity is available to cool the heated zone; the whole is kept full of water by virtue of the roof tank (assuming the pipework design has avoided traps, dead zones, etc.). The Bush Lane House system is of this general type.

Some experimental work has been done on systems having no replenishment and with no circulation of water. As evaporation occurs the water level drops and with it the area of steel protected. To achieve any useful period of fire resistance a column must project above the level required for carrying its load so that the loss of water occurs above the critical level. Careful control of steam venting is necessary to prevent significant amounts of water being carried out with the steam. A number of refinements have been proposed that assist circulation of water within an isolated column to ensure that all the water is used, but the improvements do not overcome the basic weakness of this system, unless forced circulation is used. A variant of this system, in which pumps are provided to give good circulation within the large diameter columns, has been installed in Centre Pompidou in Paris.

A refinement of the non-circulatory system with no replenishment is a non-circulatory system in which all columns are connected to a roof tank so that evaporation losses can be made good whilst steam is vented from each column.

A further variant has been used for the SNCF Offices in Paris. In this case, the two storey high columns are connected direct to the water mains, so that in the event of fire, a stop cock is turned and water flows through the columns and is discharged to waste.

Heat transfer considerations

Assessments of the fire resistance of structural elements normally follow the well-established furnace test procedures on individual elements. It will be clear that assessments of the effectiveness of water-cooled elements by such a technique are fraught with difficulties, not least because the fire protection is provided by a system involving a large part of the building. In view of this, effort has been put into developing a theoretical basis for predicting the performance of a particular system without testing each one.

As already stated, the first 'modern' work was carried out by Seigel[1]. More recently a detailed method has been published by CONSTRADO[2]. The development of the Bush Lane House system occurred between these publications, and has many similarities.

Whether the steel is on the inside or outside of the building, heat will be transferred to it by radiation and convection. If the steel is outside, the heat transfer from emerging flames will be significant. The laws of heat transfer are of course very well-established and in the context of water-filled steel the most important variables are the flame and fire temperatures and the duration of exposure. It has long been recognized that the exposure of structural elements outside buildings is less than that of internal elements, but an agreed method of assessing this has not been available. Now, following various research programmes, such a method has been devised by us on behalf first of CONSTRADO and now the American Iron and Steel Institute. The results will be published soon, but none of this has been available for use on the Bush Lane House project, nor is it incorporated in the CONSTRADO report on water-cooled steelwork.

In the absence of this more rigorous approach, the fire exposure of water-cooled external steel has been assessed by using temperatures and durations derived from the standard time-temperature curves given in standards for furnace-tested elements. The fire resistance period required by the GLC bye-laws or the Building Regulations is used to determine the duration and from the standard time-temperature curve a temperature appropriate to this is established. In the case of Bush Lane House, the temperature taken was that corresponding to the end of the one hour period, this period being that required for a conventional structural element. This was obviously a conservative assumption when used to compute the total heat transfer to the steel during a one hour fire.

Unlike other methods of structural fire protection, the performance of a water-cooled system will generally depend on the extent of the fire in the building as well as on the severity of the fire. In a circulating system all the water may be involved and so the heat from a fire on several floors must be absorbed by the same water as that from a fire on just one floor. In the design of the system an assumption must be made on the extent of fire, and this will be related to the way the building is compartmented. In Bush Lane House, each floor is a compartment floor, but as part of the conservative approach adopted, the water-cooling system was sized for a two-floor fire.

Although it is necessary to check the maximum steel temperatures by considering a fire at the bottom where steel thickness tends to be greater and water temperatures can be higher (due to the higher boiling point under increased pressure), the worst condition is a fire at the top floor. Once any reservoir is exhausted the water level in the system would drop below the level of the top horizontal pipework of the circulation system, circulation would cease and the system would rapidly fail as the top columns lost their water filling. Lower down, although water circulation would cease, the remaining water would provide some protection.

The general procedure can therefore be summarized as follows:

(a) Establish the period of fire resistance required

(b) From the standard time-temperature curve determine the maximum fire temperature

(c) Establish the assumed extent of fire for design purposes

(d) Using heat transfer theory (see CONSTRADO method[2]), calculate the maximum steel temperature for a fire low down in the building

(e) Calculate the water storage capacity for replenishment considering a fire at the top of the building.

With the new methods developed for CONSTRADO and AISI the procedure will become:

(a) Calculate fire duration

(b) Calculate effective fire and flame temperatures

(c) Calculate heat transfer to the steel

(d) Establish the assumed extent of fire for design purposes

(e) Using heat transfer theory calculate the maximum steel temperature for a fire low down in the building

(f) Calculate the water storage capacity for replenishment considering a fire at the top of the building.

Detailed considerations applying to Bush Lane House

Prior to the construction of Bush Lane House, water cooling had only been used for the protection of vertical columns. Its use for beams raises considerable difficulties in ensuring that adequate controlled water flow occurs and no steam pockets develop. Apart from its use to protect the columns and the inclined lattice members, the water cooling on Bush Lane House also protects the critical top horizontal member of the lattice and the 'ladder' members between the paired columns.

The lattice structure was fabricated in panels, which also provided a means of constraining the water flow into preferred circulation patterns, whilst minimizing site joints through which water had to flow. By using cast nodes for the lattice as well as cast tubes, it was possible to ensure that unobstructed water-flow could occur up each lattice panel. No water flow across the bolted connections between panels was necessary except in the top boom, where special precautions were needed. Generally, site connections which had to contain the water, for example the mid-height joint between lattice panels, were site-welded.

By constraining the water flow into the panels of the lattice a more ordered water circulation pattern could be ensured, with less chance of freak patterns that could lead to some members becoming filled with steam in the event of fire. It was also necessary to ensure that the flow in the top member would be consistent and that sections of it could not be by-passed. This was achieved by making the top member an incomplete ring. It connects all the tops of lattice and column panels and at one end is joined to a downpipe located in the protected space behind the lift core. At the bottom this feeds into a loop pipe within the plant room, from which feeder pipes connect into the bottom of each lattice panel.

In the event of a fire, local heating of the lattice will occur near the fire source. The heated water will tend to rise and once sufficient heating takes place, water circulation begins to occur. Initially this circulation will not involve the whole system but will take place between heated and unheated parts of the lattice, for example, between one side of the building and another. As more of the floor becomes involved in the fire, the circulation pattern will change until it becomes the overall flow in which the downward flow only takes place within the protected downpipe. This is obviously a simplified picture as many complex intermediate patterns could develop.

A critical part of the Bush Lane House system is the top horizontal lattice member. This is a highly stressed lattice component and by virtue of its position above the top windows it needs full protection from fire on the top floor. In the early stages consideration was given to having the top loop of the water cooling system separate from the lattice and placing it on the roof. Apart from difficulties this gave for positioning the track for the window cleaning cradle, it failed to solve the problem of protection for the top lattice member. However, by designing the circulation system as described, it was clear that the exposure of the main lattice members would be sufficient to produce circulation of water in the top member, which would then ensure that it remained full of water.

In fully developed fires, sufficient heat will be involved to lead to steam formation. Initially any steam formed will tend to condense in cooler parts of the system, but eventually steam/water mixture will reach the top of the lattice. Separation of the steam takes place as the mixture flows through the top of the main columns, where steam vents disperse the steam over the roof.

Loss of water is made good by the main header tank, which feeds directly down a protected shaft to the loop pipe in the plant room. The cold water is thus introduced into the cool part of the cycle. The tank system also functions to maintain the normal water level in the lattice.

In Bush Lane House the 'worst' fire conditions, for which particular checks must be made in design, were for fires at the bottom and top floors, the latter being much the most severe. At the bottom the wall thickness of the columns is greatest and so the highest steel temperatures could occur. At the top, the problem was to show that water circulation would be established satisfactorily. In practice it is likely that the major driving force for circulation will only come when some steam is formed and the rapid density decrease in the heated zone provides the necessary effect. This was satisfactory as it is only after steam is formed that circulation is essential to ensure low enough steel temperatures.

Approvals

The relevant approving authority for Bush Lane House is the Greater London Council. As the GLC has its own Scientific Branch, technical appraisal of the proposed system was carried out within the GLC, without reference to the Fire Research Station.

Although consideration was given to the idea of carrying out an *ad hoc* fire test, it was apparent that a meaningful test would be very difficult to set up. It has already been pointed out that the performance of the fire protection depends on the system as a whole, rather than on its individual parts. Before any detailed planning for such a test was carried out the GLC decided that in view of the results obtained in the tests that had been done, particularly the one in Germany in which a furnace was built around one column in the partially completed building, no test would be required. The approval would be based on theoretical considerations of heat transfer and on the details of the water circulation system.

The calculations submitted showed that the maximum steel temperature (lower floor where the columns were of greatest thickness, 36 mm plus a 4 mm slag layer internally) would be 350°C. The assumptions used were that the column would be engulfed in flame (i.e. a configuration factor of 1.0); that the fire behaved as a black body radiator (i.e. it had an emissivity of 1.0); that the stainless steel would be smoke-blackened giving good heat absorption (i.e. it would have an emissivity of 0.85); that the slag layer would have a low thermal conductivity (5.8 W/m°C, compared with 20.9 W/m°C for stainless steel); and that the fire temperature would be at the one hour value of the standard time-temperature curve for the full one hour fire duration. These are conservative assumptions, but the result showed that a considerable margin of safety still existed. Local overheating would not be a problem if water remained in the system at all points.

With these systems it has become accepted that water storage should be provided to keep the system full for the required period. No reliance is placed on topping up by the fire brigade. The calculations for reservoir capacity showed that 6 m³ of water was necessary to provide protection for a two compartment (i.e. a two floor) fire of one hour duration at the intensity stated above, including water provision for a safety margin of a further 15 minutes. The assumptions used were the same as those for the maximum steel temperature, except that the configuration factor was reduced to 0.5 as it was excessively conservative to assume, on top of all the other assumptions, that all the steel on two floors would be totally engulfed in flame for the full hour. The average heat transfer intensity to the exposed parts of the structure was calculated to be 59,000 W/m². For a one compartment fire the time taken to boil the water would be approximately $1\frac{1}{4}$ hours. For a two compartment fire it would be $\frac{1}{2}$ hour.

In discussion with the GLC, one point raised was the effect of fires involving foamed plastics in furniture where fire temperatures can rise rapidly to values well above those assumed. As radiation is proportional to the fourth power of the temperature (in absolute units) this might be significant. However, it was shown that although these high temperatures do occur, their duration is short, and that the relative high continuous temperatures assumed in the calculations were still likely to be conservative.

After further checks to ensure that the sizes of the tubes and piping were adequate after maximum water flow rates at their narrowest points, approval was given.

Other details

The solution

Although the system has been referred to as a water cooling system, the water used has to be treated to prevent freezing and to inhibit corrosion in the pipework. The chemicals used to deal with these problems are the same as those used on most other water cooled buildings, namely potassium carbonate to lower the freezing point and potassium nitrite to inhibit corrosion. 25% of potassium carbonate depresses the freezing point to about −12°C; 0.85% potassium nitrite in conjunction with the carbonate at a pH of about 10 is an effective inhibitor.

Other chemicals could have been used, but these have been found to be effective and economical, although their cost is a significant item in the cost of the system. Other treatments that may be relevant in particular circumstances are oil films to stop evaporation and bactericides to prevent growths.

It is important to limit the concentration of trace elements in the water, particularly chlorides in stainless steel tubes. As the mains water showed levels that were close to acceptable limits, demineralized water was used for Bush House Lane.

Filling

The filling procedure needs to be carefully considered. The cost of the solution precludes draining to waste if draining down is necessary for any reason. The concentration of chemicals needs to be as uniform as possible.

For Bush Lane House approximately 24,000 gallons of solution was required. This was mixed at the works of the chemical supplier and delivered to site in six tanker loads, each load being chemically analyzed.

The solution was pumped into the system via the roof tanks. After filling it was artificially circulated by using pumps connected into the main ring at plant room level.

Prior to filling with solution, the whole system had been filled with water containing a dye as a final check for water tightness. No leaks were found in the stainless steel structure.

Thermal expansion

In addition to considerations of the effects of thermal expansion on the lattice structure, the effect on the solution and on the associated pipework had to be checked.

The water level fluctuates under normal cyclic temperature variations and this variation is taken up in the space at the top of the columns and in the break tank on the roof. It was not considered practical to accommodate the volumetric expansion of solution that would occur under fire conditions. The system will overflow onto the roof and this has been allowed for in the estimates of water volumes required.

Thermal movement of non-structural pipework has been catered for by the provision of movement joints.

Maintenance

Water cooling systems are designed to be low-maintenance installations. However, at this stage in their development they cannot be considered to require no maintenance at all. Apart from the obvious point of monitoring for leaks, the main item is to check the concentration of chemicals in the solution.

Potassium carbonate is a stable material and the concentrations used are nowhere near the solubility limit. Hence no change in concentration would be expected. Inhibitor concentrations can change, particularly under bacterial action. It is therefore necessary to check this.

Maintenance will be carried out annually. The solution will be circulated by means of the plant room pumps and samples taken from the tops of the columns.

Conclusion: Future applications

Although the methods for the detailed design of liquid-filled structures are now established it is unlikely that very extensive use will be made of this method of fire protection. The technology involved is not particularly sophisticated, but workmanship standards must be of a very high order. The steel fabrication requires process plant standards rather than normal structural standards if the welds are to remain watertight under the high static pressures involved.

The need for special approvals can be another inhibiting factor. The obtaining of such approvals should become easier as more 'case law' becomes established, but that is probably many years away.

It is likely that the most appropriate applications will be those where the additional pipework associated with the system can be kept to a minimum, i.e. tall buildings rather than long, low ones. Under these conditions the cost of the system can be significantly less than the cost of protection by conventional casing and then the extra complications can be worthwhile.

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Acknowledgements

John Campbell and Alan Denney carried out much of the detailed work associated with the water cooling system for Bush Lane House.

Credits

Client:

City & West End Properties Ltd.

Architects, Engineers & Quantity Surveyors:

Arup Associates

Main Contractor:

Trollope & Colls

Vibrations of rigid foundations

Ari Danay

Introduction

In some practical cases the design of foundations for machinery only marginally involves dynamic analysis. Some machines, such as turbo-generators, do not in principle exert harmonic (or oscillating) forces or couples, though dynamic loads may arise briefly when the machine is loaded abruptly or there is a short-circuit. However, in general, dynamic loads do occur during normal machine operations, and may arise from the following sources:

(a) Parts which are not perfectly balanced about the axis of rotation, as in turbines, centrifugal pumps, turbo-generator sets and fans

(b) Internal combustion engines, piston-type compressors and pumps, steam engines and other machinery involving a crank mechanism, which may develop unbalanced forces due to the conversion of the reciprocating motion into rotary motion. These forces may act at the frequency of rotation, in which case they are called primary components, or at double this frequency, when they are referred to as secondary components.

(c) Vibratory conveyors, used to transport masses of solid particles, which are effectively designed as mass-spring oscillators (trough-leaf-springs) activated by a motor-crank-drive mechanism tuned to the natural frequency of the system

(d) Punch presses, forging hammers, drop tests, stamping machines, etc. which produce impulsive loads that may be considered as single pulses because the effect of one load dies out before the next occurs.

The first three types of loads may usually be represented by the product of an equivalent unbalanced mass and eccentricity with the square of the operating frequency for (primary components) or its double, for secondary components.

The above mentioned loads are usually specified by the manufacturer who, in some cases, may also supply an adequate foundation design for 'average' soil conditions.

The basic design criteria for machinery foundations are specified in *CP2012* (1974) and, in more detail, in reference books like Richart and Hall[10]. Briefly, two fundamental criteria should be satisfied:

(a) That resonance does not occur between the frequencies of the pulsating loads and the natural frequencies of the foundation/soil system

(b) That the amplitude of any vibration does not exceed safe limits.

Whenever possible, the highest disturbing frequency should be lower than half of the foundation/soil natural frequency, otherwise higher than twice. For installations of lesser importance the first frequency ratio may be increased to 0.6 and the second reduced to 1.5. Where the machine is connected to the foundation block by means of low frequency, resilient anti-vibration mountings, the frequency ratio should be higher than 3.

The amplitude limits may be classed as follows:

(a) To avoid damage to machinery

(b) To avoid damage to buildings

(c) To avoid discomfort to persons

(d) To avoid settlement of foundation.

The first limit is usually set by the manufacturer, and where he does not discharge this responsibility the others are taken to be sufficient. The second and the third depend strongly on the disturbing frequency and vary between 2 and 200 microns (1 micron = 10^{-6}m). The last is less clear, but is generally considered to be 200 microns for most soil types. For loose sands and silts with a high water table the safe limit will probably be unknown and it may be necessary to dodge the problem by consolidating the soil or by piling down to a more satisfactory soil. All the above limits should be divided by a safety factor, largely dependent on the refinement of calculations and the reliability of soil data. Usually the factor is taken as 1.5.

To predict the foundation behaviour under dynamic loading, some knowledge of the soil properties, such as the composition, elastic modulus, Poisson's ratio and bulk density, should be obtained, either by laboratory or in situ tests, or both. For preliminary design, installations of lesser importance with clearly defined load and soil parameters or where the above limits are satisfied beyond any reasonable doubt, the soil investigation may be simplified to mere classification and choice of the soil subgrade coefficients as set by Barkan.[5]

Design also involves the bearing pressure due to dead and live loads, the ground water level and existence of rigid strata near the surface, the effects of transmission of heat from the machine to the foundation block, anti-vibration mountings and fixing of the machine to the foundation block, concrete properties, construction procedure and reinforcement requirements to minimize shrinkage cracking.

DYNAMIC ANALYSIS

Simplified lumped parameter model

To comply with the frequency ratio and vibration amplitude limits some dynamic analysis should be carried out. Each relevant degree of freedom (rigid body motion) has an associated generalized load, mass moment of inertia, equivalent viscous damper and soil spring stiffness coefficient. Assuming that the block and the machine are infinitely rigid and knowing their geometry, centres of gravity and mass moments of inertia about the main axes, the maximum number of degrees of freedom is reduced to six. Due to symmetry of loading and geometry this number is usually lower. Some of these motions may be coupled (i.e. are not independent of each other), as in the following cases:

(a) Vertical and rotational displacements where the rigid block and machine centre of gravity is not vertically above the resultant bearing pressure

(b) Horizontal displacements and rocking or pitching rotations when the lateral ground pressure, horizontal load and block plus machine centre of gravity are not in the same line.

In the preliminary analysis this coupling is usually ignored. When it is small enough (its measure being the eccentricity ratio to the relevant block dimensions), it may also be ignored in the final design. Whether dealing with a one or multi-degree of freedom model, the basic component is a mass-spring-damper model consisting of the following parameters:

(a) Rigid machine or foundation block, defined by its centre of gravity and mass moment of inertia

(b) Elastic spring, representing either the soil elastic properties or the anti-vibration mountings, defined by the associated spring stiffness

(c) Viscous damper, representing energy losses in the soil or anti-vibration mounting, defined either directly by the damping coefficient or by the critical damping ratio

(d) Generalized load, e.g. force, moment or twisting couple, defined by amplitude and frequency.

These parameters will be denoted as follows:

M^j = mass moment of inertia
K_s^j = spring stiffness
C_s^j = damping coefficient
P^j = load amplitude
and Ω^j = load frequency, in radians/sec.

The index j = 1 ... 4 refers to each of the following degrees of freedom:

(1) Vertical motion
(2) Horizontal motion
(3) Rocking or pitching rotation
(4) Yawing rotation (i.e. twisting about a vertical axis).

If we use the tonf, metre, second system, we shall have the following units:

For j = 1, 2; M tonf-sec²/m = W/9.81, K tonf/m,
C tonf-sec/m, P tonf.

and for j = 3, 4; M tonf-m-sec² = Iw/9.81, K tonf-m/rad,
C tonf-sec-m/rad, P tonf-m.

with W = weight (tonf), $Iw = \int \gamma c^2 \, dV$ where 'γ' is the specific weight (tonf/m³), 'c' is distance from axis of rotation and the integration is taken over the whole volume of the block (or machine).

For soils, the calculation of the spring stiffness is usually based on an equivalent circular soil contact area, defined by radius r. Since usually the contact area is not circular, this radius is obtained from the requirement that the area or the second or polar moment of area of the soil contact is identical to that of the base. The equivalent radius will thus depend on the degree of freedom being considered.

For a rectangular shape, with sides 2a (parallel to X axis) and 2b (parallel to Y axis), this requirement results in the following relations:

(1) Vertical and horizontal motion: $r = 2\sqrt{\frac{ab}{\pi}}$

(2) Rotation about X: $r = 2\left(\frac{ab^3}{3\pi}\right)^{\frac{1}{4}}$

(3) Rotation about Y: $r = 2\left(\frac{a^3b}{3\pi}\right)^{\frac{1}{4}}$

(4) Yawing (rotation about Z): $r = 2\left(\frac{ab(a^2+b^2)}{6\pi}\right)^{\frac{1}{4}}$

For an arbitrary shape:

(1) Vertical and horizontal motion: $r = \sqrt{\frac{A}{\pi}}$

(2) Rotation about X: $r = \left(\frac{4}{\pi}\int y^2 dA\right)^{\frac{1}{4}}$

(3) Rotation about Y: $r = \left(\frac{4}{\pi}\int x^2 dA\right)^{\frac{1}{4}}$

(4) Yawing (rotation about Z): $r = \left(\frac{2}{\pi}\int (x^2+y^2) dA\right)^{\frac{1}{4}}$

where the integrations are taken over the whole of the contact area A.

Table 1

Coefficients C_1, C_2, C_3 and C_4 and stiffness K_s from Veletsos and Verbic[8]

Motion	K_s	n	Poisson's Ratio	C_1	C_2	C_3	C_4
Horizontal			0	0	0	0	0.775
(j = 1)	$\frac{8Gr}{2-\mu}$	$\frac{8}{2-\mu}$	$\frac{1}{3}$	0	0	0	0.65
			$\frac{1}{2}$	0	0	0	0.60
Vertical			0	0.25	1.0	0	0.85
(j = 2)	$\frac{4Gr}{1-\mu}$	$\frac{4}{1-\mu}$	$\frac{1}{3}$	0.35	0.8	0	0.75
			$\frac{1}{2}$	0	0	0.17	0.85
Rocking			0	0.8	0.525	0	0
(j = 3)	$\frac{8Gr^3}{3(1-\mu)}$	$\frac{8}{3(1-\mu)}$	$\frac{1}{3}$	0.8	0.5	0	0
			$\frac{1}{2}$	0.8	0.4	0.027	0

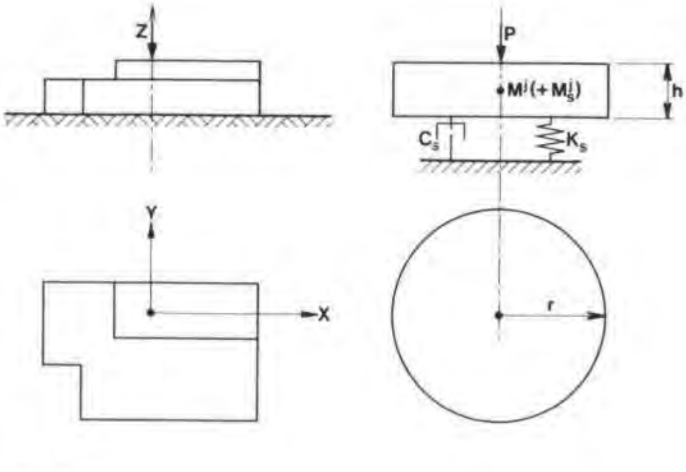


Fig. 1

The expressions for the static stiffness K_s^j are given in Table 1 and Fig. 1, as a function of r, and of G and μ, the shear modulus and Poisson's ratio for the soil.

The internal damping coefficient for soils is usually calculated from an equivalent critical damping ratio, defined by:

$$D \text{ (damping ratio)} = C_s \Big/ 2\sqrt{K_s M} \quad \text{......(1)}$$

Recommended values are given in Table 2, from various sources.

Table 2

Type of soil	Equivalent D	Reference
Dry sand and gravel	0.03–0.07	Weissmann and Hart (1961)
Dry and saturated sand	0.01–0.03	Hall and Richart (1963)
Dry sand	0.03	Whitman (1963)
Dry and saturated sands and gravels	0.05–0.06	Barkan (1962)
Clay	0.02–0.05	Barkan (1962)
Silty sand	0.03–0.10	Stevens (1966)
Dry sand	0.01–0.03	Hardin (1965)

The natural circular frequency of the simple oscillator, defined by:

$$\omega = \sqrt{(K_s/M)} \text{ rad/sec} \quad \text{......(2)}$$

should comply with the frequency ratio requirements mentioned in the introduction, i.e.

Preferably $\Omega/\omega < 0.5$ (or 0.6); else $\Omega/\omega > 2.0$ (or 1.5 or 3)...(3)

If these relations are satisfied, the vibration amplitudes are calculated as:

$$U_{max} = U_{st} \times DLF \quad \text{......(4)}$$

where the equivalent static displacement, $U_{st} = P/K_s$ ......(5)

and $DLF = \text{dynamic load factor} = \frac{1}{\sqrt{((1-R^2)^2 + 4D^2R^2)}}$ ......(6)

and:

$$R = \text{frequency ratio} = \Omega/\omega \quad \text{......(7)}$$

The above model, with the natural frequency and displacement limitations, may ensure a safe design for many cases in which dynamic computations are required. However, it should be borne in mind that this model is only a simplification of the soil dynamic behaviour, the main aspects neglected being:

(a) Some volume of soil underneath the rigid base is also set in motion, resulting in an 'added' or 'virtual' mass which should be considered in the equilibrium equation.

(b) The elastic waves, generated by the vibrating base, are not entirely reflected by the underlying soil layers. The resulting energy loss is quantified by an equivalent damping coefficient, usually referred to as 'geometric' or 'radiation'.

If the base natural frequency was determined according to the second of Equation 3 (i.e. the base is low-tuned), the neglect of the above properties will have been on the conservative side since the true natural frequency will be lower than calculated.

The opposite effect may occur if the first of Equation 3 was satisfied (high-tuned), in which case the frequency ratio is shifted towards resonance, but the increased damping ratio is likely to compensate for it in the DLF calculation (Equation 6).

Frequency-dependent soil characteristics

As mentioned above, the problem we are dealing with is of elastic waves propagating from the rigid base through the soil medium.

The overall solution of the half-space displacements and stresses is of no direct interest here, but the resulting equation of motion of a point underneath the base (usually the projection of the centre of gravity) is used to determine the above parameters, namely added soil mass (M_s) and damping coefficient (C_s).

Surface disc on elastic half-space

To simplify the mathematical model, the base is idealized by an equivalent rigid disc of radius r and mass M. The elastic, homogenous and isotropic soil medium is represented by the shear modulus G, Poisson's ratio μ, and density ρ.

Early solutions of this problem were given by Reissner[1], who assumed a linear pressure distribution underneath the base. Quinlan[2] and Sung[3] studied the effect of parabolic and 'rigid base' stress distributions, assuming these to be constant throughout the range of frequencies considered. Bycroft[4] evaluated the weighted average of displacements beneath the footing and Hsieh[5] reorganized Reissner's equations to improve the presentation of added mass and radiation damping coefficient. Due to simplifying assumptions all the above solutions were limited to only a low range of frequencies. Luco and Westmann [6] and Veletsos and Wei[7] extended the solution beyond this range and presented results in diagrammatic or tabular form. A simplified model, neglecting coupling terms between rocking and sliding motion, was presented by Veletsos and Verbic[8] in simple algebraic equations, including also the internal damping in soil.

Simple design formulae, based on the early solutions (limited to a low range of frequencies), are given by Hall[9] and used in Richart, Hall and Woods[10] manual. The purpose of this paper is to extend the applicability of these formulae in view of the latest analytical solutions and provide also a set of design diagrams.

Denoting an undimensional frequency parameter by:

$$a = \Omega r \sqrt{\rho/G} \qquad (8)$$

the frequency dependent added soil mass and damping coefficient may be expressed by:

$$M_s = \frac{\rho r^2}{G a^2} K_s \,(1 - F_1(a)) \qquad (9)$$

$$C_s = r \sqrt{\left(\frac{\rho}{G}\right)} K_s \, F_2(a) \qquad (10)$$

where K_s is the static stiffness of the soil as in Table 1. For j = 1, 2, 3, Veletsos and Verbic[8] expressed the functions $F_1(a)$ and $F_2(a)$ by:

$$F_1(a) = 1 - \left(\frac{C_1 C_2^2}{1 + C_2^2 a^2} + C_3\right) a^2 \qquad (11)$$

$$F_2(a) = C_4 + \frac{C_1 C_2^3 a^2}{1 + C_2^2 a^2} \qquad (12)$$

where $C_1 \ldots C_4$ are coefficients given in Table 1.

The functions $F_1(a)$ and $F_2(a)$ for j = 4 (yawing motion) may be calculated from the compliance functions f(a) and g(a) shown in Fig. 2 taken from Luco and Westman's[6] paper with:

$$F_1(a) = \frac{f(a)}{((f(a))^2 + (g(a))^2)} \qquad (13)$$

$$F_2(a) = \frac{1}{a} \; \frac{g(a)}{((f(a))^2 + (g(a))^2)} \qquad (14)$$

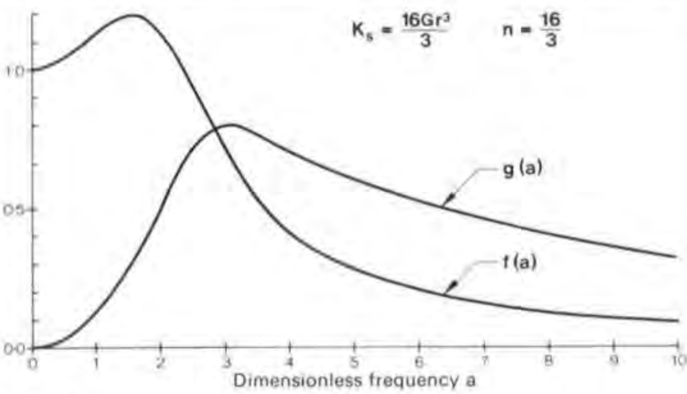


Fig. 2

Compliance functions f(a) and g(a) from Luco and Westmann[6].

If we define the following undimensional quantities:

$$n = K_s/Gr \qquad (15)$$

$$b = M/\rho r^3 \qquad (16)$$

$$b_s = M_s/\rho r^3 \qquad (17)$$

for horizontal and vertical motion, where M is mass of base and ρ is soil density.

Table 3

Comparison between Veletsos and Verbic[8], Hsieh[5] and Hall[9] formulations

Quantity	Poisson's Ratio	Veletsos and Verbic[8]			Hsieh[5] (for $a \leq 1.5$)			Hall[9] (for $b \geq 1$)		
		Horizontal	Vertical	Rocking	Horizontal	Vertical	Rocking	Horizontal	Vertical	Rocking
b_s (added soil mass ratio)	0	0	$\frac{1}{1+a^2}$	$\frac{0.588}{1+0.276a^2}$	0.2	0.5	0.4	0	0	0
	$\frac{1}{4}$	—	—	—	0.2	1.0	—	0	0	0
	$\frac{1}{3}$	0	$\frac{1.344}{1+0.64a^2}$	$\frac{0.8}{1+0.25a^2}$	—	—	—	0	0	0
	$\frac{1}{2}$	0	1.36	$0.144 + \frac{0.683}{1+0.16a^2}$	0.1	2.0	—	0	0	0
D (radiation damping ratio)	0	$\frac{0.775}{\Psi}$	$\frac{0.85+\frac{0.25a^2}{1+a^2}}{\Psi}$	$\frac{0.0945}{(1+0.276a^2)\Psi}$	$\frac{0.561+0.07a}{\Psi}$	$\frac{0.825+0.1a}{\Psi}$	$\frac{0.122a}{\Psi}$	$\frac{0.616}{\sqrt{b}}$	$\frac{0.85}{\sqrt{b}}$	$\frac{0.245}{(1+0.375b)\sqrt{b}}$
	$\frac{1}{4}$	—	—	—	$\frac{0.57+0.068a}{\Psi}$	$\frac{0.953+0.173a}{\Psi}$	—	$\frac{0.631}{\sqrt{b}}$	$\frac{0.981}{\sqrt{b}}$	$\frac{0.283}{(1+0.281b)\sqrt{b}}$
	$\frac{1}{3}$	$\frac{0.712}{\Psi}$	$\frac{0.918+\frac{0.219a^2}{1+0.64a^2}}{\Psi}$	$\frac{0.1a^2}{(1+0.25a^2)\Psi}$	—	—	—	$\frac{0.639}{\sqrt{b}}$	$\frac{1.041}{\sqrt{b}}$	$\frac{0.3}{(1+0.25b)\sqrt{b}}$
	$\frac{1}{2}$	$\frac{0.692}{\Psi}$	$\frac{1.202}{\Psi}$	$\frac{0.059a^2}{(1+0.16a^2)\Psi}$	$\frac{0.606+0.087a}{\Psi}$	$\frac{1.22}{\Psi}$	—	$\frac{0.665}{\sqrt{b}}$	$\frac{1.202}{\sqrt{b}}$	$\frac{0.346}{(1+0.188b)\sqrt{b}}$

Note: $\Psi = \sqrt{(b + b_s)}$

and

$$n = K_s/Gr^3 \quad (18)$$
$$b = M/\rho r^3 \quad (19)$$
$$b_s = M_s/\rho r^3 \quad (20)$$

for rocking and yawing motion, we obtain from Equations 9 and 10:

$$b_s = \frac{n(1-F_1(a))}{r^2} \quad (21)$$

and

$$\text{damping ratio, } D = \frac{C_s}{2\sqrt{(K_s(M+M_s))}} = \frac{F_2(a)}{2}\sqrt{\frac{n}{b+b_s}} \quad (22)$$

Table 3 shows a comparison between the expressions for b_s and D given by the Veletsos and Verbic[8], Hsieh[5] and Hall[9] formulations. With the added mass M_s, the effective natural frequency of the base will be:

$$\omega_{eff} = \sqrt{\frac{K_s}{M+M_s}} = \omega\sqrt{\frac{M}{M+M_s}} = \frac{\omega}{\sqrt{(1+b_s/b)}} \quad (23)$$

and the frequency ratio:

$$R_{eff} = \Omega/\omega_{eff} = R\sqrt{(1+b_s/b)} \quad (24)$$

where ω and R are the frequency and frequency ratio calculated simply from Equations 2 and 7.

Substituting Equation 24 in Equation 6 yields:

$$DLF = \frac{1}{\sqrt{((1-R^2(1+b_s/b))^2 + 4D^2R^2(1+b_s/b))}} \quad (25)$$

Most soil materials have a shear modulus in the range 1000 to 5000 tonf/m². With an average density of 0.18 tonf/sec²/m⁴, the undimensional frequency 'a' lies in the range $0.006\,\Omega r$ to $0.0134\,\Omega r$.

Considering a possible load frequency range of 20 to 400 rad/sec approximately 200 to 4000 RPM) and a radius of 1 to 5 m, the range of likely values of 'a' is $0.12 \leq a \leq 27$.

Denoting the base thickness by h metres, writing $\alpha = h/r$, and taking the density of concrete as ρ_c = 0.24 tonf-sec²/m⁴ and that of soil as

ρ = 0.18 tonf-sec²/m⁴ the base mass ratio may be expressed by:

$$b = \frac{\rho_c \pi r^2 h}{\rho r^3} = \frac{\rho_c \pi \alpha}{\rho} \text{ for translational displacements,}$$

$$b = \frac{\rho_c \pi r^2 h\,(r^2/4 + h^2/3)}{\rho r^5} = \frac{\rho_c}{\rho}\,\frac{\pi\alpha}{12}\,(3+4\alpha^2) \text{ for rocking,}$$

$$\text{and } b = \frac{\rho_c \pi r^4 h}{2\rho r^5} = \frac{\rho_c \pi \alpha}{2\rho} \text{ for yawing.}$$

By substituting the numerical values of ρ_c and ρ and by taking 'α' to be restricted to the range $0.2 \leq \alpha \leq 2$ we have the following ranges for 'b':

Translations: $0.8 \leq b \leq 8$

Rocking: $0.2 \leq b \leq 13$

Yawing: $0.4 \leq b \leq 4$

The approximate formulations of Hsieh[5] and the further simplifications of Hall[9] are based on the assumption that commonly the value of a lies between 0 and 1.5 and b is larger than 1. It can be seen from the above ranges of values that this assumption is rather restricting.

Embedment and internal damping

In many common cases, the base is either fully or partially embedded in ground or its stiffness is supplemented by 'dowels' or short piles. Though the local stresses beneath are obviously different from those of the elastic half-space formulation, many investigators suggest that for practical purposes the overall dynamic behaviour is not substantially altered (see Kansel[11]). If an increased stiffness can be determined (Refs. 12 to 16) then we write:

$$\varphi = K_{se}/K_s \quad (26)$$

where K_{se} is the stiffness of the embedded base and replace b and b_s in Equations 16, 17, 19 and 20 by

$$B = M/\rho r^3\varphi \quad (27)$$
$$B_s = M_s/\rho r^3\varphi \quad (28)$$

or

$$B = M/\rho r^5\varphi \quad (29)$$
$$B_s = M_s/\rho r^5\varphi \quad (30)$$

then Equations 21 to 25 still hold with B and B_s replacing b and b_s.

In the previous formulations for an elastic half-space the internal soil damping was neglected. Veletsos and Verbic[8] have shown that it can be included in the elastic half-space theory by means of the parameter tan δ defined by: (see Fig. 3)

$$\tan\delta = \frac{\Delta W}{2\pi W} \quad (31)$$

where: ΔW = energy loss per cycle of vibrations of a material with hysteretic behaviour (area of the elliptical shear stress-strain diagram) and W = the strain energy stored in an equivalent perfectly elastic material (shaded area).

Laboratory tests on soil samples have shown that the ratio in Equation 31 is practically independent of the load frequency but is a function of the amplitude of the strain involved. This assumption, which represents the damping by a constant $\Delta W/W$ ratio, is called a constant hysteretic model and will be used in the following:

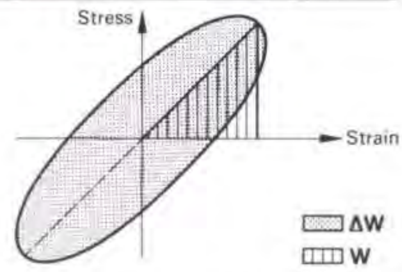


Fig. 3

The dynamic equilibrium equation of the base may be written as:

$$K_s u + C_s \dot{u} + (M+M_s)\ddot{u} = P \sin \Omega t \quad (32)$$

By substituting in Equation 32 the expressions for K_s (Table 1), M_s and C_s from Equations 9 and 10, taking the Fourier Transform and replacing G by a complex shear modulus

$$G^* = G(1 + i\tan\delta),\ i = \sqrt{-1} \quad (33)$$

and finding the inverse Fourier Transform, Veletsos and Verbic[8] obtained:

$$M_s = \frac{\rho r^2}{Ga^2} K_s(1 - F_1^*(a)) \quad (34)$$

$$C_s = r\sqrt{\frac{\rho}{G}} K_s F_2^*(a) \quad (35)$$

where:

$$F_1^*(a) = F_1(a) - a F_2(a)\sqrt{(0.5(\sqrt{(1+\tan^2\delta)} - 1))} \quad (36)$$

$$F_2^*(a) = F_2(a)\sqrt{(0.5(\sqrt{(1+\tan^2\delta)} + 1))} + \frac{F_1(a)\tan\delta}{a} \quad (37)$$

The functions $F_1(a)$ and $F_2(a)$ are given in Equations 11 and 12.

DESIGN GRAPHS

On the basis of the previously described Veletsos and Verbic[8] formulae for added mass, radiation damping coefficient and inclusion of internal damping, Figs. 4 to 12 have been prepared. These show $(M + M_s)/M$, DLF and DLF × R² as functions of $R = \Omega/\omega$ for R between 0.2 and 10 and of B for B between 0.2 and 20. B is calculated from Equations 27 or 29, or, where embedment effects are neglected, B = b from Equations 16 or 19.

The first function can be used to calculate the effective frequency ω_{eff} from Equation 23.

The last function, DLF × R², refers to the relative amplitude of motion of machines with unbalanced masses. Denoting by M_uL the sum of the products of eccentric masses with their eccentricities, the load may be expressed by:

$$P(\Omega) = M_uL\,\Omega^2 \sin\Omega t \quad (38)$$

The dynamic amplitude of motion will therefore be:

$$A = \frac{P(\Omega) \times DLF}{K_s} = \frac{M_uL\,\Omega^2 \times DLF}{M\omega^2} = \frac{M_uL\,R^2 \times DLF}{M} \quad (39)$$

The diagrams were calculated using a value of tan δ = 0.06 (which is similar to 3% viscous damping ratio*) and for Poisson's ratios of 0.33 and 0.5. As seen from Equations 36 and 37, the effect of the internal damping, besides increasing the overall damping coefficient, is to alter the added mass function $F_1(a)$. It can be seen from the diagrams that the tendency of the frequency-dependent added mass and radiation damping is to smooth out the peaks of the response so that in some systems the phenomenon of resonance can be hardly said to occur.

The graphs use as reference the natural frequency ω of the mass-spring system, calculated without the added mass of the soil.

If the expressions for K_s and B are substituted in Equation 2, we obtain:

$$\omega = \frac{1}{r}\sqrt{\frac{G}{\rho}}\sqrt{\frac{n}{B}} \quad (40)$$

And, substituting Equation 40 in Equation 8:

$$a = \frac{\Omega}{\omega}\sqrt{\frac{n}{B}} = R\sqrt{\frac{n}{B}} \quad (41)$$

Since the values of n lie between 3.33 and 8 (see Table 1), values of 'a' exceeding 1.5 (limit of validity of Hsieh[5] formulae) may occur either for frequency ratios greater than 1 and heavy bases (e.g. B>1) or with low frequency ratios and light (or embedded) bases.

By substituting in Equation 40 the expressions for B and n and the mass moment of inertia of cylindrical bases, we obtain:

$$\omega = \frac{2V_s}{r}\sqrt{\frac{\varphi}{\alpha}}\,C_f \quad (42)$$

where: $V_s = \sqrt{\frac{G}{\rho}}$ = ground shear-wave velocity, φ = embedment ratio (of stiffness), α = h/r and C_f = a function of Poisson's ratio and soil density ρ, given in Table 4 on page 27.

*Note: with $B_s = 0$, $F_1 = 1$ and $F_2 = 0$, and using Equations 22 and 37:

$$D = \frac{\tan\delta}{2a}\sqrt{\frac{n}{B}} = \frac{\tan\delta}{2\Omega r}\sqrt{\frac{Gn}{\rho B}} = \frac{\tan\delta}{2}\left(\frac{\omega}{\Omega}\right)$$

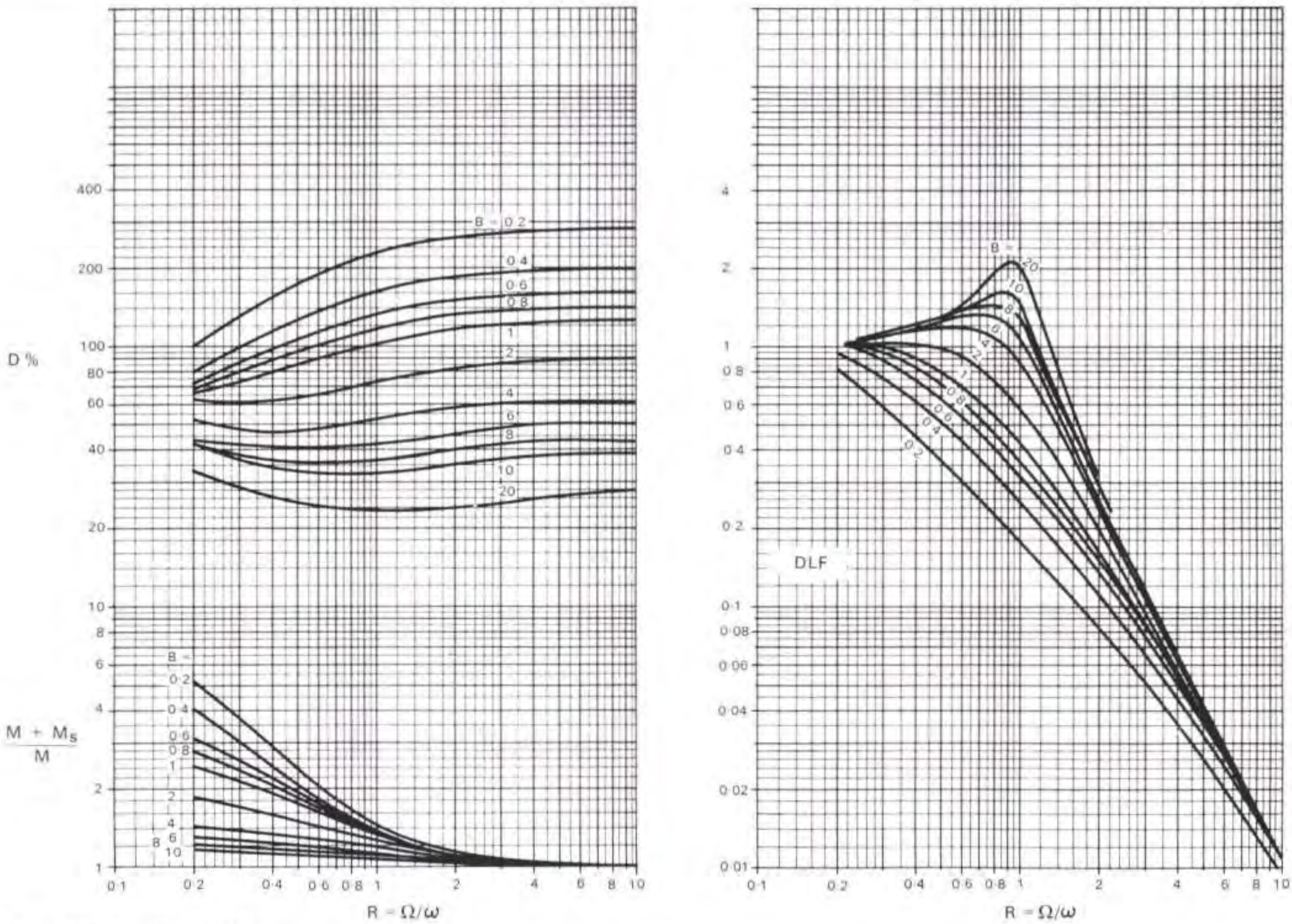


Fig. 4
Mass ratio, damping ratio and DLF
Vertical vibration; $\mu = \frac{1}{3}$

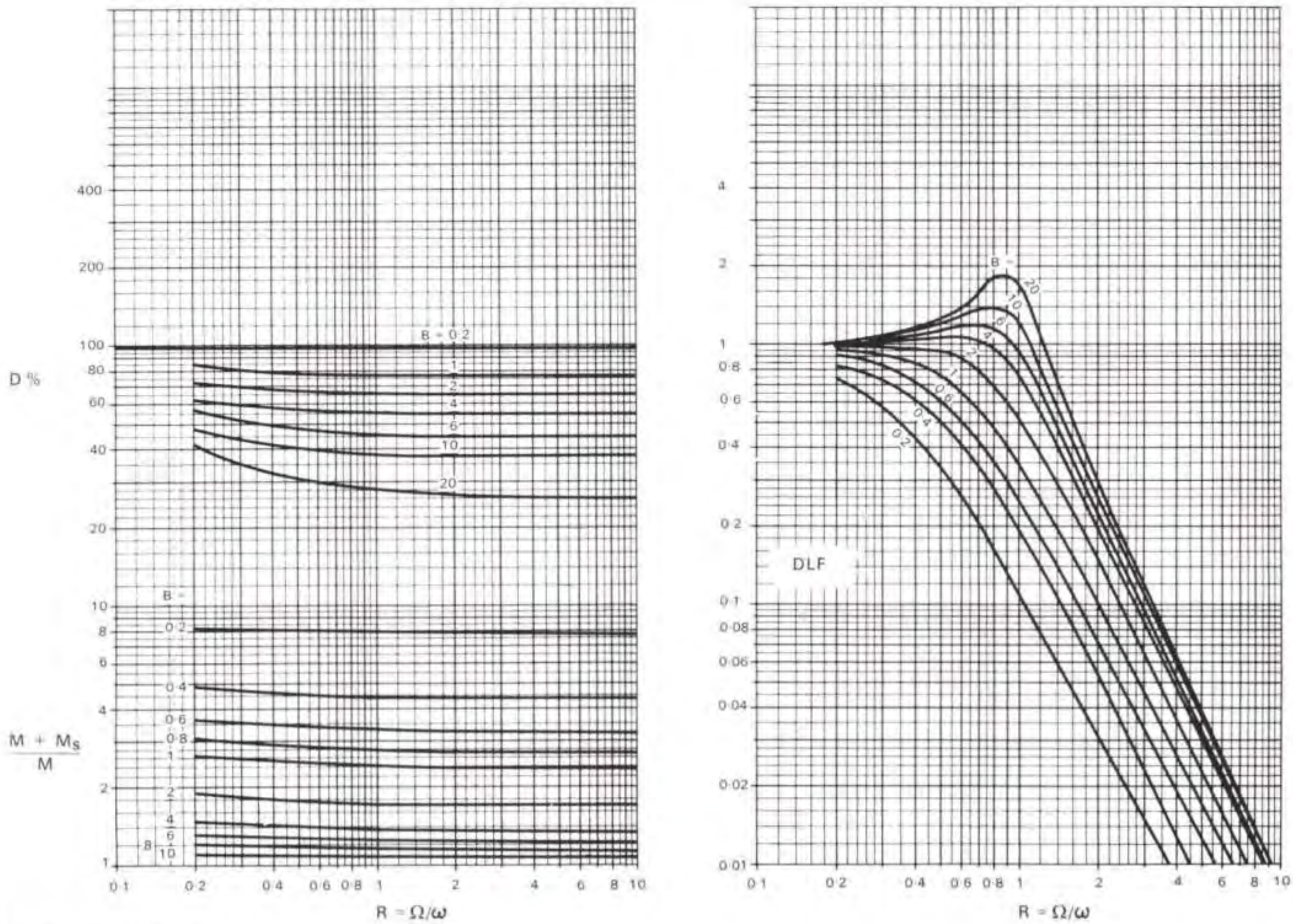


Fig. 5
Mass ratio, damping ratio and DLF
Vertical vibration; $\mu = \frac{1}{2}$

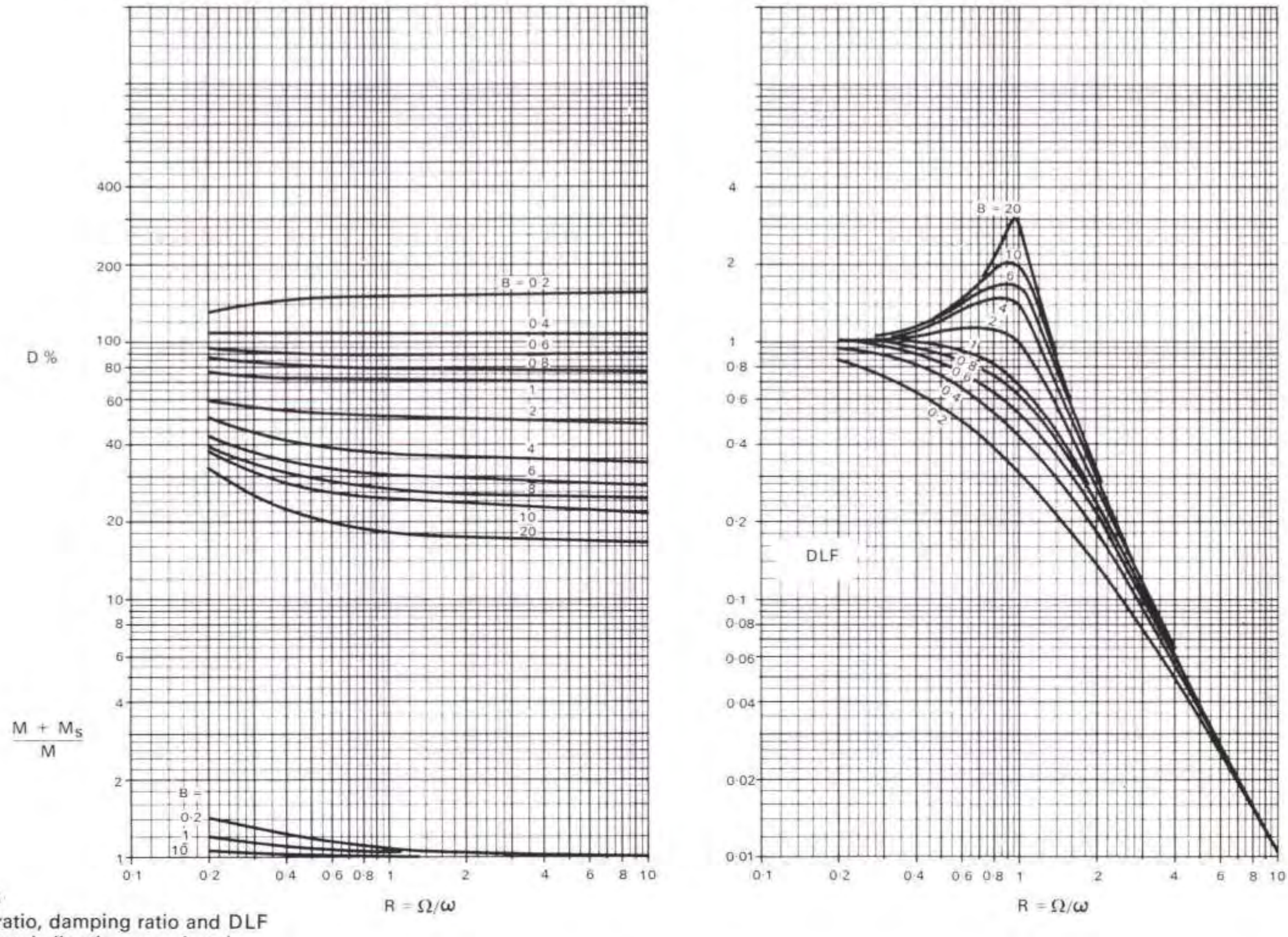


Fig. 6
Mass ratio, damping ratio and DLF
Horizontal vibration; $\mu = \frac{1}{3}$ or $\frac{1}{2}$

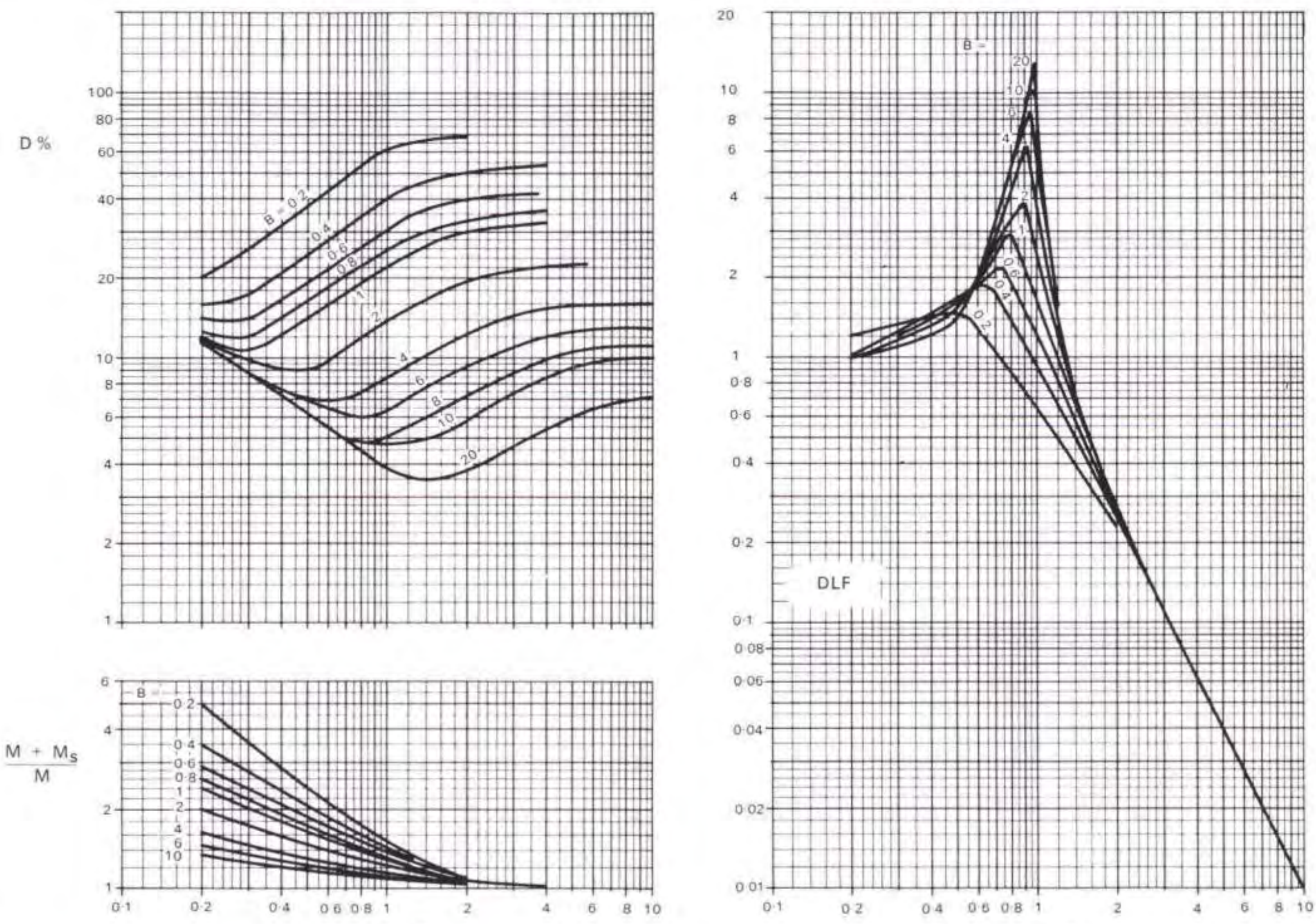


Fig. 7
Mass ratio, damping ratio and DLF
Yawing vibration; all μ

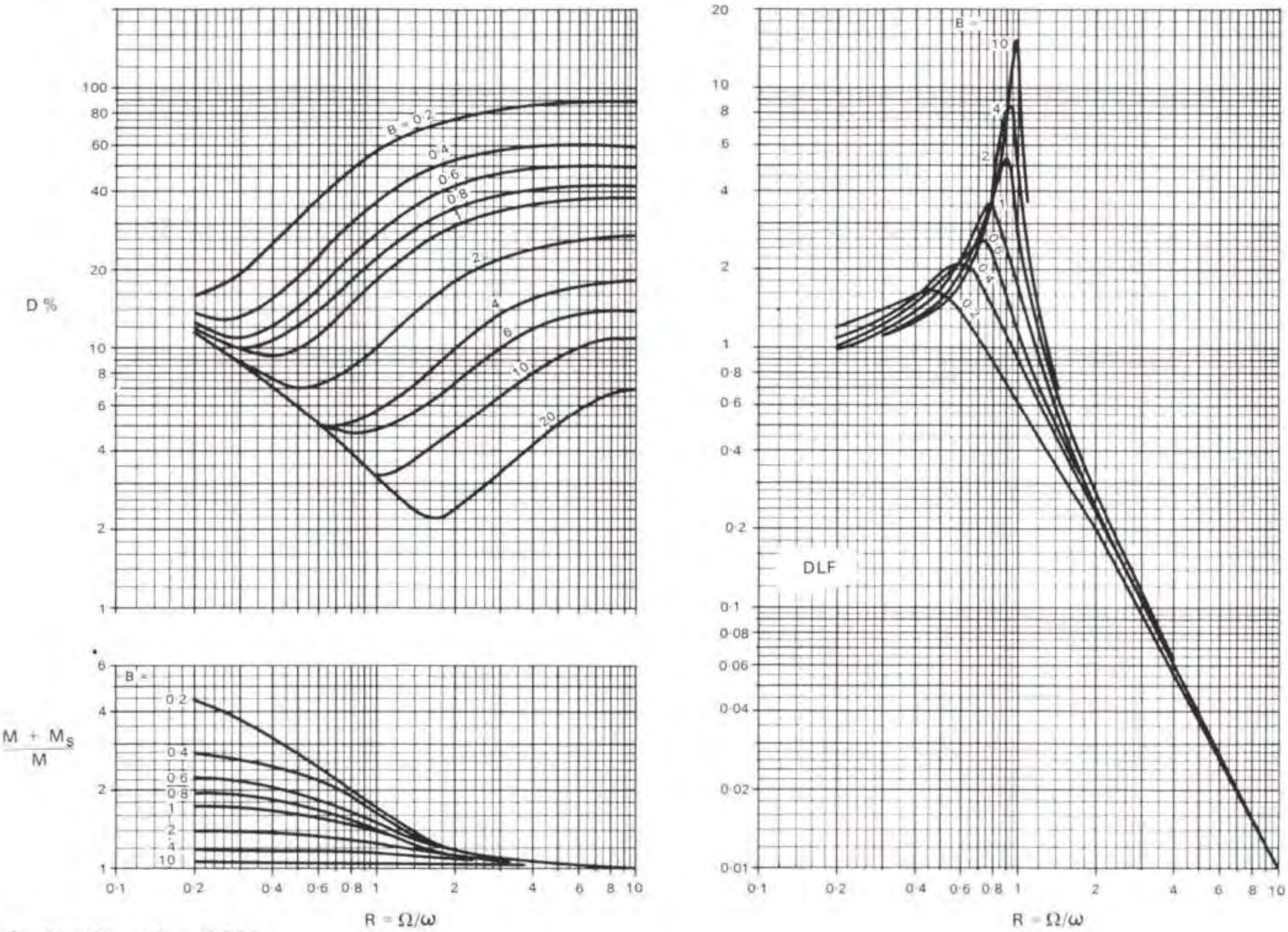


Fig. 8
Mass ratio, damping ratio and DLF
Rocking vibration; $\mu = \frac{1}{3}$

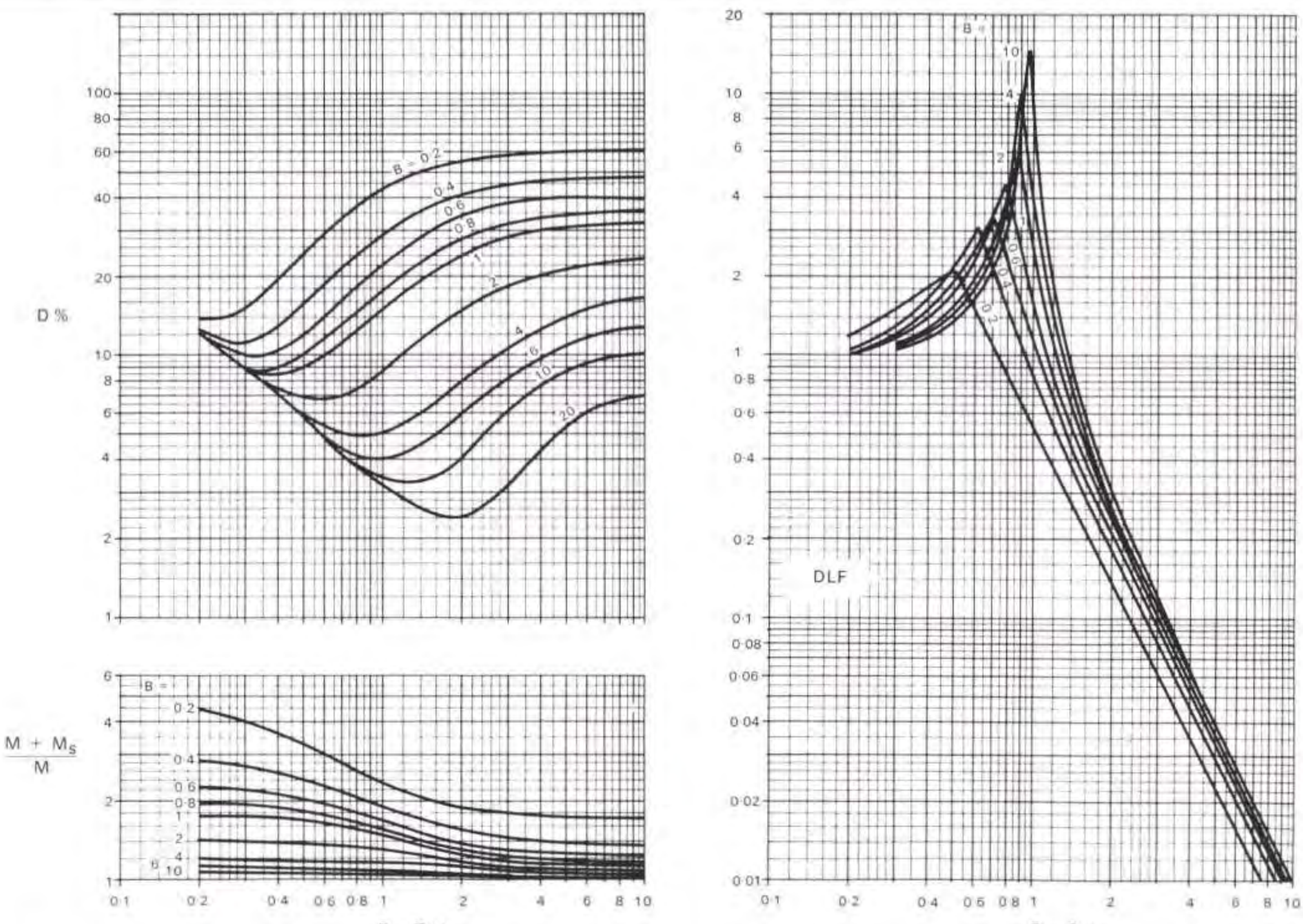


Fig. 9
Mass ratio, damping ratio and DLF
Rocking vibration; $\mu = \frac{1}{2}$

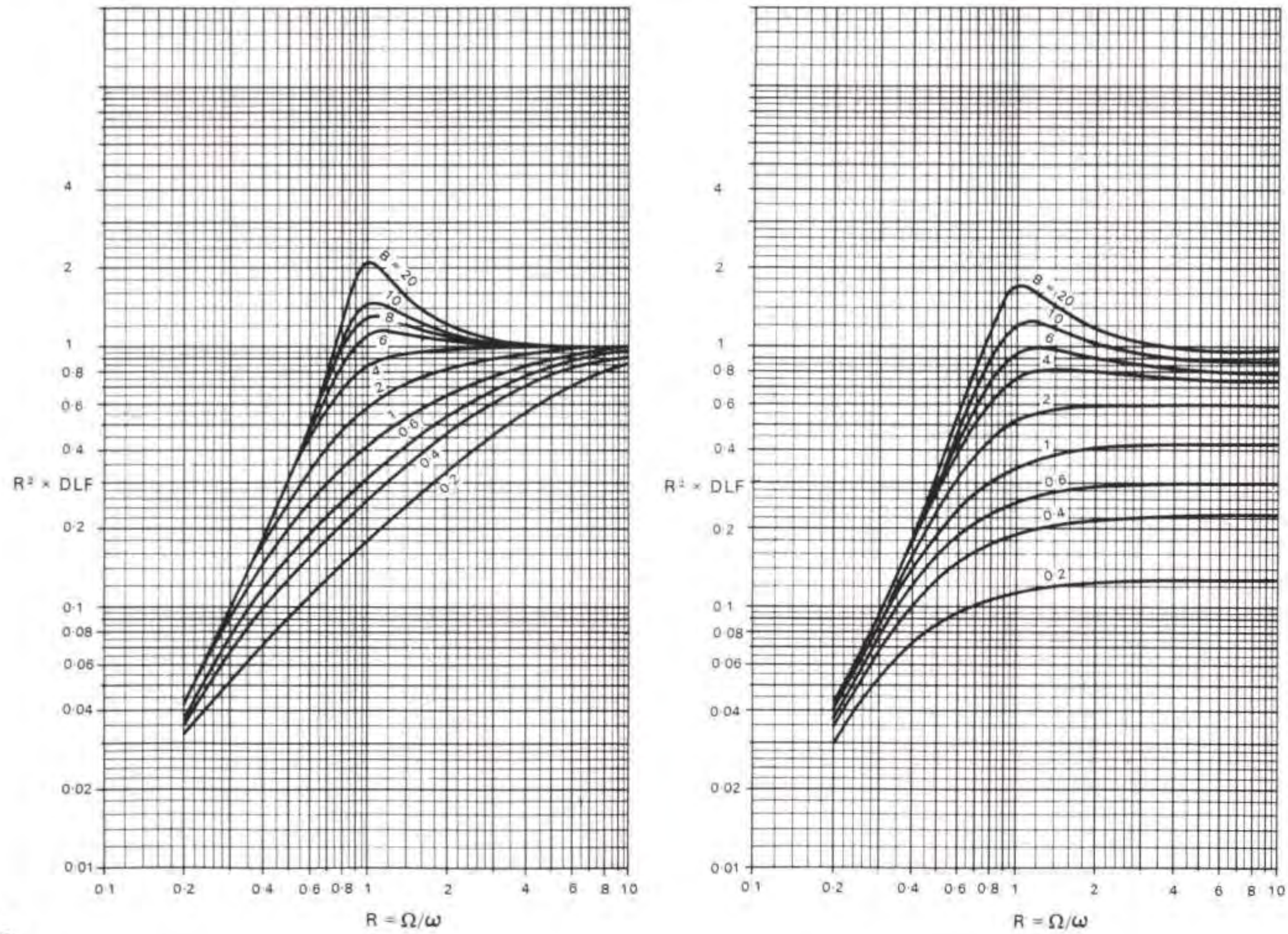


Fig. 10
$R^2 \times DLF$
Vertical; $\mu = \frac{1}{3}$.

Vertical; $\mu = \frac{1}{2}$

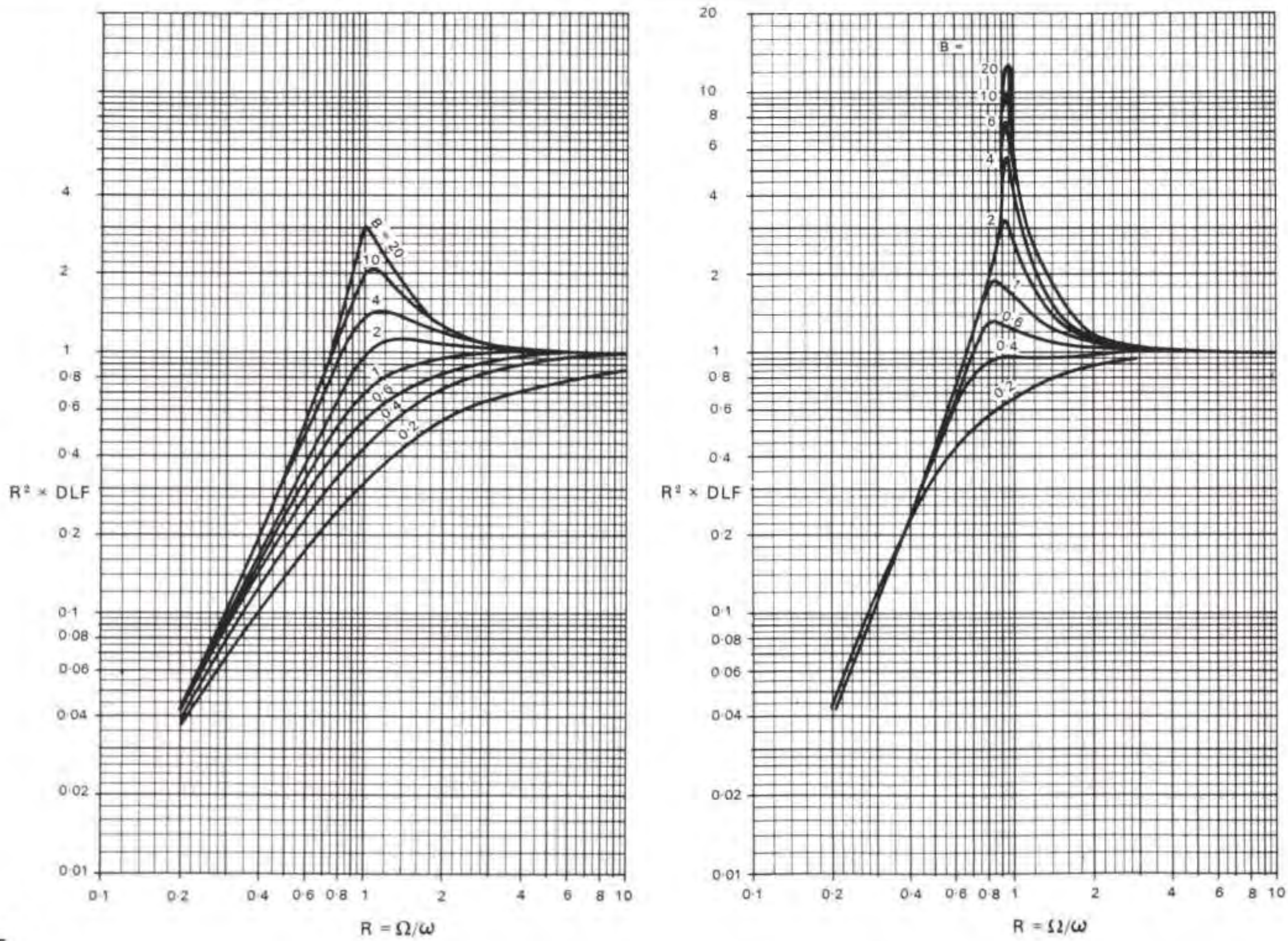


Fig. 11
$R^2 \times DLF$
Horizontal; $\mu = \frac{1}{3}$ or $\frac{1}{2}$.

Yawing; all μ

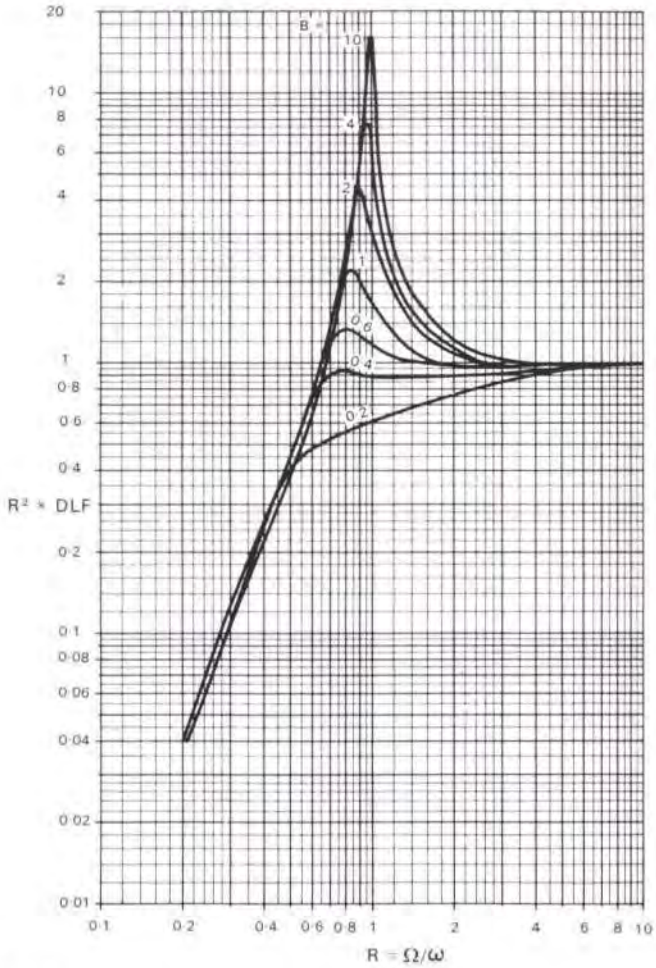
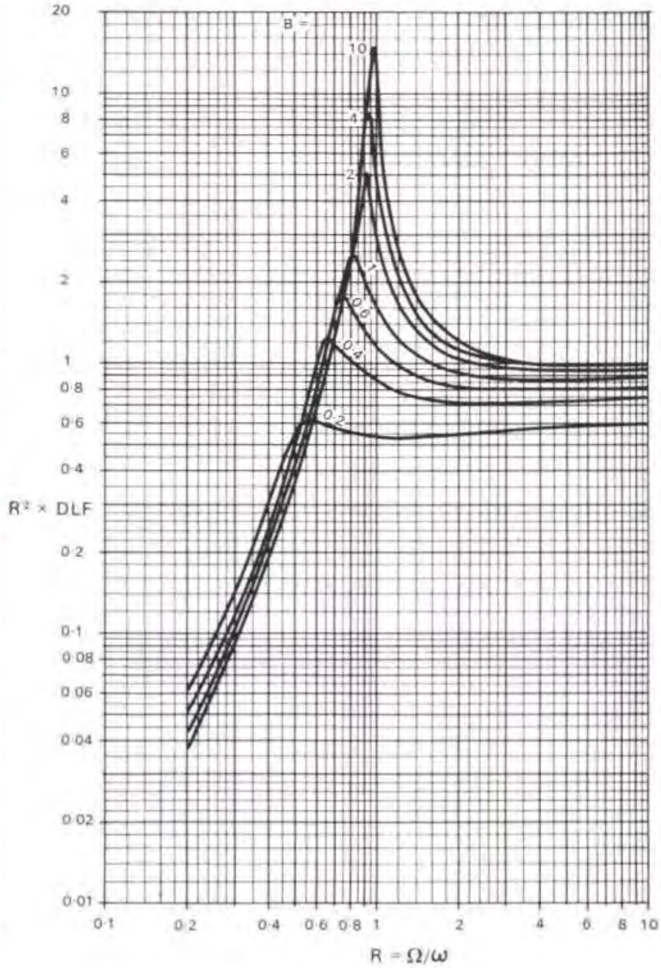


Fig. 12
$R^2 \times DLF$
Rocking; $\mu = \frac{1}{3}$.



Rocking; $\mu = \frac{1}{2}$

Table 4

Motion	$K_s\varphi$ Stiffness	M Mass	Cf
Vertical	$\frac{4Gr\varphi}{1-\mu}$	$\rho_c \pi r^3 \alpha$	$\frac{1}{\sqrt{(\pi\lambda\,(1-\mu))}}$
Horizontal	$\frac{8Gr\varphi}{2-\mu}$	$\rho_c \pi r^3 \alpha$	$\sqrt{\frac{2}{\pi\lambda\,(2-\mu)}}$
Rocking	$\frac{8Gr^3\varphi}{3\,(1-\mu)}$	$\frac{\rho_c \pi r^5\,(3\alpha + 4\alpha^3)}{12}$	$\sqrt{\frac{8}{\pi\lambda\,(1-\mu)\,(3+4\alpha^2)}}$
Yawing	$\frac{16Gr^3\varphi}{3}$	$\frac{\rho_c \pi r^5 \alpha}{2}$	$\sqrt{\frac{8}{3\pi\lambda}}$

$$\lambda = \frac{\rho_c\ (\text{concrete})}{\rho\ (\text{soil})}$$

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